Cognitive Limitation in Complex Decision-Making: Evidence from Centralized College Admission

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Abstract

In centralized admission systems, applicants submit a rank-order list (ROL) to indicate their preference ordering. The optimal decision rule is often complex and entails an accurate understanding of strategic considerations, contingent reasoning, and backward induction. We study college admissions in Ningxia, China, to understand how students construct their lists. The admission system employs a constrained Deferred Acceptance Algorithm to match students to colleges. Applicants are allowed to choose four colleges out of hundreds of alternatives. We find that applicants are overly cautious with their top choices and the majority of them do not always put safer choices at a lower-ranked spot on the list. We propose a boundedly rational decision rule to explain such choices. This decision rule proposes that applicants myopically focus on the spot they are contemplating and neglect its impact on the rest of the list. To differentiate our hypothesis from other explanations, we exploit variations in priority scores to test choice patterns that are robust to heterogeneous preferences, and collect additional evidence with a survey that tests predictions of the model related to framing effects. A mixture model with both rational and behavioral types fits well the administrative data and yields better out-of-sample predictions than the one-type rational model. Estimates of our preferred specification classify around 50% of the sample as the behavioral type, and the share is higher among the socioeconomically disadvantaged. In the counterfactual analysis, de-biasing leads to a substantial welfare gain for behavioral applicants and also for the majority of rational types by enhancing matching efficiencies. In terms of both equity and efficiency, de-biasing performs better compared to merely switching to other mechanisms such as unconstrained Deferred Acceptance or a Boston mechanism in which the bias could be less costly.

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1 Introduction

Centralized admission systems play an important role in school-student matching around the world. In these systems, students are assigned to schools based on the outcome of a matching mechanism that accounts for students' preferences (typically called rank-order lists, or ROLs) and their priority scores. In most of the mechanisms, determining the optimal ROL requires significant strategic sophistication on the part of the student. Constructing the optimal list is a risk-reward tradeoff problem that requires backward induction and contingent reasoning, concepts that applicants may have trouble grasping.[1]

We study college choices in Ningxia, China, to empirically investigate how student applicants respond to an example of such mechanisms. The centralized admission system in Ningxia employs a constrained Deferred Acceptance Algorithm, where eligible students can list up to four colleges from 239 first-tier colleges. The algorithm works by considering students' demand in order of their scores from the College Entrance Exam (CEE). When the algorithm reaches a given student, it considers the student's first-choice college and assigns the student to this choice if the admission quota of the college has not been filled. The algorithm checks the next choice only when the choice in question has already been filled by higher-scoring students, and repeats this process until the fourth choice. In practice, the student is relegated to a second-tier college outside of the 239 choices if all of the listed colleges have already been filled.

Undoubtedly, any student needs to manage the risk of relegation to a second-tier college by listing at least one safe first-tier college on her list. The amount of risk that the student should take for the other three choices, however, is less obvious. Intuitively, the cost of not getting into one's first-choice college is less devastating than not getting into one's fourth choice, because all four choices are still in the top tier. The best college for any rank on the student's list depends on her choices for the next rank down. This intuition prescribes that rather than viewing each spot in isolation, the student should formulate a contingent plan to make the most of the entire portfolio by backward induction.

Institutional features in this empirical setting make it ideally suited to analyze the decision-making of students. Colleges always prefer students with higher CEE scores to fill their quotas, and they only need cutoff scores to dictate their admission decisions, so that the number of admitted students meets the quota. Students know their priority scores by the time of application, as well as information about the cutoffs in the past, which are stable with cross-year correlation around 0.95. The stability of cutoffs not only implies that it is relatively easy for students to estimate their admission probability, but also that the utility of admission is strongly negatively correlated with admission probability, highlighting the tradeoff between them.

[1] Chade and Smith (2006) and Shorrer (2019) theoretically characterize and develop algorithms for constructing the optimal portfolio in stylized settings of college admission, both of which invoke dynamic programming thinking that is similar to the logic of backward induction. Calsamiglia et al. (2020) shows that backward induction solves computationally intractable problems for a wide class of mechanisms that are used in real college admission systems. Camerer et al. (1993) and Johnson et al. (2002) document failure of backward induction in extensive form games. Martínez-Marquina et al. (2019) documents how uncertainty impedes contingent reasoning.

The data source for the present study is an administrative dataset that records the rank-order lists, admission outcomes, and students' home address at district level, which proxies their socioeconomic status. Importantly, by enlisting the help of local high schools, we were able to conduct an incentivized online survey among 15% of the applicants who applied for first-tier colleges at the end of July 2020, right after they finished their applications and submitted their ROLs. The survey questions elicit more information about students' decision-making process, and help validate our hypothesis about students' behavior.

How do students actually respond to the mechanism? We find that the vast majority of the students choose colleges with little risk for their fourth choices. Risk taking for their first choices, however, is highly heterogeneous, with 25% of the students choosing a safe college, whose unconditional probability[2] is larger than 88.5% even for their first choices. In other words, many students appear not to be selective enough in their top choices. Meanwhile, more than 55% of the lists exhibit what we call "risk-taking reversals" (that is, within any pair of listed colleges, the probability of admission to the higher-ranked exceeds the probability of admission to the lower-ranked). Under the assumption that students' preferences largely align with college selectivity, this phenomenon is contrary to the qualitative prediction of rational benchmarks, and is present in other developing countries which use similar mechanisms in their school choice systems (Lucas and Mbiti, 2012; Ajayi, 2013). Further calibration exercise suggests that student applicants seem to be too cautious with their first choices, and exhibit too many reversals in their lists.

These behaviors correlate with demographics, contributing to inequality in admission outcomes. As detailed in Section 4.3, socioeconomically disadvantaged students are substantially more cautious in their first choices and are more likely to exhibit risk-taking reversals anywhere on the list, further deviating from the rational benchmark. Conditional on their priority scores, disadvantaged students end up in colleges whose quality, as measured by competitiveness, is more than 0.1 standard deviation lower than their advantaged counterparts.

In other contexts such as taxation, health insurance, and social program take-up,[3] agents respond to the most available part of the problem but ignore/under-react to the others when a decision problem is complex. Anecdotal evidence from college-application advisory platforms on the Chinese Internet suggests that the most common mistake in the application process is to treat each spot on the list as separate and identical, ignoring the interdependence between different spots. Such neglect resembles how human beings react to complexity in other contexts.

To the best of our knowledge, relatively limited progress has been made in understanding suboptimal strategies in the presence of such complexities.[4] Pathak and Sönmez (2008) pioneer

[2] The unconditional probability is the probability of meeting the admission cutoff of a college.

[3] Examples in taxation include Chetty et al. (2009); Congdon et al. (2009); Abeler and Jäger (2015); Rees-Jones and Taubinsky (2020). Examples in aid application include Dynarski and Scott-Clayton (2006). Examples in health insurance include Brot-Goldberg et al. (2017). Examples in social benefits take-up include Dynarski and Scott-Clayton (2006); Bertrand et al. (2006); Bhargava and Manoli (2015).

[4] Some progress has been made in understanding mistakes when the situation is straightforward strategically. Artemov et al. (2017) posits that, in a strategy-proof environment, the mistakes on the lists are primarily inconsequential. Shorrer and Sóvágó (2018) find that, in Hungary, college application dominated choices are more likely to be made when expected cost is lower and argue that multiple imperfections in decision-making may contribute

this line of work by proposing that, in the Boston mechanism, some applicants may ignore the strategic aspect of the mechanism and declare their preference ordering truthfully. The complexity of constructing the optimal list, however, is still formidable when agents are aware of the strategic aspect of a mechanism but lack the ability to maximize their expected utility using backward induction and contingent reasoning. Lack of such ability is conceptually different from preference-based or belief-based explanations, and may systematically affect how agents behave in different ways than predicted by other frictions.

To fill in this gap, we introduce a boundedly rational decision rule inspired by a Directed Cognition Model (henceforth DC) (Gabaix et al., 2006) that can naturally fit these patterns. In our context, the DC model predicts that students focus their cognition entirely on the single spot they are contemplating (i.e., they choose a college to maximize improvement of expected utility for a portfolio that consists of that spot and a subjective, perhaps psychological, outside option) and ignore the impact of this choice on the rest of the ROL. This decision rule reduces a portfolio choice problem to repeated discrete choice problems, dispensing with backward induction. Moreover, it generates a set of predictions that differentiates the rule from one that is driven by other frictions.

Prediction 1, **"More Cautiousness Only at Top"**, says that the DC Type takes substantially less risk for their first choices. However, their fourth choices are similar to their rational counterparts in terms of risk taking. This helps explain our previous observation that, under the rational rule, the overall cautiousness is too much for the first choice, but not for the fourth choice.

Prediction 2, **"Risk-Taking Reversals"**, states that the DC Type is more likely to rank riskier colleges at a lower position in ROLs, at times generating a dominated choice in which the applicant ranks a lower-quality college higher on her list of choices, while ranking a higher-quality college lower. This helps explain our previous observation that, under a rational rule, the substantial presence of risk-taking reversals seems to be incompatible with the stability of cutoffs.

Importantly, these two predictions are made under the assumption that students' preferences align with the selectivity of colleges (i.e., vertical preferences). Distinguishing the DC Decision Rule from its rational counterpart when preferences are horizontal – i.e., preferences that do not completely align with competitiveness – is a vital task for our empirical analysis. To that end, we test two more predictions of our model that differentiate our hypothesis from most alternative considerations, including horizontal preferences.

Prediction 3, **"Framing Effect"**, indicates that the DC Type takes more risks if the ROL problem is transformed to mathematically equivalent lottery questions. We analyze the incentivized questions from the online survey to test this prediction. Estimates using our preferred specification suggest that 48.7% (SE=1.8%) of the students in our survey sample are classified as the DC Type. Moreover, echoing our findings in the administrative data, a 1 SD increase in our socioeconomic

to this. Dreyfuss et al. (2019) proposes that Koszegi-Rabin expectation-based reference dependence may explain the dominated choice in Li (2017)'s experimental data. (Chen and Sönmez, 2002; Hassidim et al., 2016; Li, 2017; Rees-Jones and Skowronek, 2018; Rees-Jones et al., 2020) documents the existence of suboptimal play in the lab. Lucas and Mbiti (2012); Ajayi (2013); De Haan et al. (2015); Shorrer and Sóvágó (2018); Kapor et al. (2020) find that disadvantaged students are particularly likely to suffer from additional harm in strategy play for reasons including information friction and mistaken beliefs status index is associated with a 7.3% (SE=1.8%) decrease in propensity to be the behavioral type.

Prediction 4, **"Upward Movement"**, implies that if a student's test score counterfactually increases, any colleges listed by rational agents will only move down along the list. For DC decision-makers, however, any listed college will first move up along the list, and then move down, exhibiting a hump-shaped track on the list. To empirically test the presence of upward movement, we construct small score bins, and compute share of listed colleges within each bin to exploit small variations in priority scores that do not correlate with students' preferences. This empirical exercise identifies substantial presence of such movements.

Having established the existence of the DC type, we structurally estimate a mixture model of college choices, where both DC and Rational Type coexist, using simulated method of moments.[5] We construct moments on students' risk-taking in terms of unconditional admission probability, risk-taking reversals, and college characteristics that aim to capture preferences over colleges. Importantly, all moments are constructed for each position separately, which, in light of the last prediction, contributes to the identification of the DC type.

The mixture model fits the data better than the single-type model even when extra flexibility is corrected by BIC analogues, and it yields substantially better out-of-sample predictions relative to the single-type rational model. A closer look at the fit of a Rational Type-only model suggests that the flexibility of college preference in our model generates less cautiousness and fewer risk-taking reversals, echoing our first two predictions.

The estimated share of the DC Type is substantial, and ranges from 45.1% (SE=0.54%) to 55.1% (0.55%). The estimates are comparable to the share of the DC Type estimated from the survey. Moreover, a 1 SD increase in the socioeconomic index is associated with a decrease that ranges from 3.68% (SE=0.56%) to 6.02% (SE=0.55%) in the share of the DC Type. The curvature over college selectivity is mild or even convex, suggesting that selective colleges are highly sought after, consistent with anecdotal evidence on the prestige of more selective colleges in China.

De-biasing leads to a substantial welfare gain among behavioral applicants, which is on average larger than an increase of roughly 0.25 s.d. in the test score under the old equilibrium. Surprisingly, de-biasing also leads to a substantial, although smaller, welfare gain among the majority of rational applicants, by enhancing matching efficiencies (Abdulkadiroğlu et al., 2011). On the other hand, switching to either an unlimited list or a Boston mechanism without de-biasing, while intuitively making the bias less relevant, decreases welfare overall, echoing Chen and Kesten (2017)'s theoretical findings on the benefit of China's parallel mechanisms. A more detailed look at welfare change by students' socioeconomic status reveals that while on average de-biasing improves the welfare of students of all socioeconomic status, the disadvantaged benefit more from it. In contrast, both unconstrained Deferred Acceptance and the Boston Mechanism decrease the welfare of students of any socioeconomic status.

[5] The conventional approach in this line of literature is to maximize the likelihood of the reported ROL, as in Agarwal and Somaini (2018); Calsamiglia et al. (2020); Kapor et al. (2020). We take this alternative approach because in our setting it is computationally intractable to numerically simulate the likelihood, as the choice set is too large.

Our paper adds to the literature on behavioral mechanism design (Hassidim et al., 2016; Li, 2017; Rees-Jones and Skowronek, 2018; Dreyfuss et al., 2019). We show that in the presence of complexity, a specific decision heuristic can better explain participants' strategies. More generally, the DC decision rule reflects the difficulty of backward induction (Camerer et al., 1993) and the tendency to engage in myopic choices when cognitive limitation prevents decision-makers from aggregating choices in multiple stages under uncertainty (Rabin and Weizsäcker, 2009). Our analysis also demonstrates the benefits of structural modeling in the analysis of behavioral agents (DellaVigna, 2018).

The paper also is closely related to the recent surge of studies on empirical student-school matching. Methodologically, our paper is similar to papers that employ both administrative and survey data to test for optimal strategic play (De Haan et al., 2015; Kapor et al., 2020). Our paper further demonstrates that, despite the deviation from optimality, behavioral decision rules can be easily incorporated into the framework of revealed preference analysis laid out in Agarwal and Somaini (2018, 2019), and can help unmask the mechanisms behind the choice patterns of high-achieving disadvantaged students. Our findings suggest that certain cognitive limitations create gaps in admission outcomes among applicants of the same academic ability, and thus are related to the literature on the distributional consequences of behavioral biases (Campbell, 2016; Bhargava et al., 2017; Allcott et al., 2019; Rees-Jones and Taubinsky, 2020). While the underlying mechanism is different, this also echoes studies that cover the distributional impact of school choices in decentralized systems (Walters, 2018).

The paper proceeds as follows. Section 2 introduces the empirical setting and data sources. Section 3 lays out the problem mathematically and discusses the basic components in decision-making. Section 4 presents the evidence concerning **Cautiousness** and **Risk-Taking Reversals**. Section 5 introduces the DC Decision Rule and derives the set of predictions that differentiates it from the optimal rule. Section 6 tests **Framing Effects** using the online survey data. Section 7 tests **Upward Movement** using the administrative data. Section 8 presents results from our structural estimation. Section 9 addresses alternative hypotheses that could affect our findings. Section 10 concludes.

2 Empirical Setting

2.1 Summary of Events Before Admission Process

Figure 1 presents the timeline of the admission procedure in 2020 (the first timeline) as well as before 2020 (the second timeline). The postponement in 2020 happened because of COVID-19, and as a result all the related activities were postponed by exactly one month. Therefore, the order of and duration between admission procedures was unchanged to minimize the impact of the pandemic on student applicants and college admission committees.

Student applicants are required to take the College Entrance Exam (CEE), a nationwide exam that takes place less than one month before the start of college admission. As elaborated in Section

2.2, the exam performance determines students' priority scores in the admission system and thus has a predominant impact on students' application strategies.

The admission process (and therefore ROL) is stratified according to college quality. Before 2019, the colleges were classified into three tiers of decreasing quality: elite (first-tier), public non-elite (second-tier) and private non-elite (third-tier). The latter two categories merged starting in 2019, but the elite category remains unchanged. In this paper, we focus on the admission to elite colleges, which are argued to play a central role in upward mobility in China because of the tremendous value placed on education and the huge return in labor markets (Jia and Li, 2016).

When students are notified of their test scores and corresponding provincial rankings, the online college application system opens. At that time, students know whether their score meets the minimum requirement to apply for schools in the 1st-tier college category, which is set by Ningxia Provincial Education Authorities. The share of students who are eligible for elite colleges is roughly $20 \sim 25\%$ of the exam takers. The eligible students on the science track can choose up to four colleges from among 239 elite colleges. For those who are on the humanities track, the total number of elite colleges is 150. Science track students account for more than 80% of the first-tier applicants.

The application process is time constrained and cognitively demanding. The time to contemplate college choices is less than three weeks if a student starts to think about it right after taking the CEE, and is less than one week if effective decision-making starts after the notification of CEE scores. Applicants must submit their ROLs before the deadline. In addition to the time constraint, students have no chance to learn about this system by trial and error before submitting their final decisions, despite the novelty of this decision environment. Anecdotal evidence suggests that misunderstanding is not rare. According to several college application advisory platforms on the Chinese Internet, one of the most common mistakes is to treat the four spots on the ROL as separate and equal, effectively ignoring the order in which ROL is processed[6]. This type of mistake concurs with the DC decision rule that we propose in this paper.

2.2 Priority Score

The priority score is almost completely determined by the College Entrance Exam (CEE)[7]. The CEE is a nationwide closed-book written exam held once a year on June 7th and 8th, with the rare exception that the exam was postponed to July 7th and 8th in 2020 due to COVID-19. To apply for colleges in an admission cycle, all students must take the CEE of the same cycle. In each province, students on the same track (Humanities or Sciences) will take the same exam. As demonstrated in Figure 1, it will take up to two weeks for Ningxia Provincial Education Authorities to grade students' exams. Students will be notified of their exam score and ranking in Ningxia around the 20th-25th of the month in which the exam takes place.

[6]See here for an example of comments about mistakes in college applications (in Chinese).

[7]Exceptions include winners of international Olympiad contests, students who win sports scholarships, students with exceptional art talent, students who belong to certain minority ethnic groups, etc. These exceptions are also quantified and added to priority scores on top of the exam scores, and are observable in our datasets.

Exams for both tracks include Chinese, Mathematics[8] and English. For each of these subjects, students get an integer score, with the maximum (best) possible being 150 and the minimum being 0. Additionally, students on the Humanities Track take a comprehensive exam on history, politics, and geography, whereas students on the Sciences Track take another exam on physics, chemistry, and biology. This track-specific exam accounts for 300 points. Thus the total score of the CEE (sum of the scores from the four subjects) is 750 points. In case of a tie in total score, ranking will be determined by the score in the comprehensive exam in the respective track, Mathematics, and English, in a lexicographic way.

2.3 Rules of Admission

After the deadline for ROL submission, the centralized admission system assigns students to colleges using a deferred acceptance algorithm based on their priority score and the colleges' preannounced admission quota. Since the priority score for each student is the same for all colleges, the mechanism is effectively a serial dictatorship mechanism, where colleges only need to specify a priority score cutoff to decide which applicants are admitted, regardless of a college's position on the student's submitted ROL.

Table B1 presents examples of how cutoffs determine admission outcomes. In Example 1, the student is admitted to college A because her score exceeds A's cutoff. Therefore the system ignores all her lower-ranked choices. In Example 2, the student is admitted to college B because her score does not meet A's cutoff but exceeds B's cutoff. As a result, the system assigns her to B, ignoring C and D. In Example 3, the student is unassigned because she does not meet the cutoff of any college she listed. In Example 4, the student is assigned to college D because she does not meet the cutoffs of A,B, and C but meets the cutoff of D.

After assignment of first-tier colleges have been completed, students are notified of the admission decision within a month. Students who are not admitted to any first-tier colleges will be passed on to the next stage of admission, where the centralized system will assign them to lower-tier colleges using the same priority score and similar algorithms [9].

We can tell from the rules above that the structure of preferences on the college side is unusually simple compared to other matching markets where there is no obvious priority score (e.g., NRMP (Roth and Peranson, 1999)), or where different priority scores are adopted in different schools (e.g., New Haven School Matching (Kapor et al., 2020)). The one-dimensional priority score enables us to interpret this specific matching market in canonical contexts. Mapping this context onto bidding, for example, students, being aware of their "WTP" (priority score), can submit up to four bids to colleges, whose "prices" (admission cutoffs) are unknown by the time of "bid submission" (ROL submission).

[8] Mathematics for the Humanities Track differs from that for the Science Track.

[9] Students can rank four 2nd-tier colleges and four 3rd-tier colleges

2.4 Data

We use two sources of data. The first is the administrative dataset generated by the centralized admission system that records the application behavior of students from 2014 to 2018. The dataset is maintained by the Ningxia Provincial Education Authorities. The second is an online survey that we set up in 2020 that targets the same type of students we analyze in the administrative dataset (i.e., CEE takers in Ningxia applying to first-tier colleges).

(I) Administrative Data With the help of the Ningxia Education Authority, we obtained access to administrative data from 2014 to 2018, covering 6,000-9,000 first-tier eligible science track students in each admission cycle. The number is considerably lower for humanities track students, amounting to roughly 1,500-2,000. The data contains students' CEE scores, ROLs, admission outcomes, and some demographic information, such as the county, city and street of their residence address (available only in 2015, 2017, 2018), gender, age, ethnicity, parents' occupation, rural/urban status, and schools where they received elementary and secondary education.

(II) Online Survey In August 2020, we carried out an online survey that targeted high school students who had applied for first-tier colleges in 2020. Four local high schools actively encouraged students to take our survey. As a result, while any first-tier eligible applicants in Ningxia could respond to our online survey, our sample mainly consists of students from these four high schools. We were able to collect 1,412 complete and effective responses, roughly 15% of the total number of first-tier college applicants in 2020. As shown in Figure 1, the survey was conducted right after students submitted their ROL for first-tier colleges, but before they were notified of the admission outcomes[10]. The survey consisted of three parts. The first part elicited basic information, such as their final ROL, high school, gender, age, parents' education and occupation, CEE score, source of application advice, and preferences over college characteristics. The second part elicited their beliefs about unconditional admission probability[11] of the colleges on their ROL ($0\% \sim 100\%$) and how satisfied they would feel (0-100) if they were admitted to a particular college. The third part consisted of several incentivized risk-taking questions that were presented in the form of a ROL and lottery, which are discussed in detail in Section 6.1.

[10]We choose this particular timing for three reasons. First, in order to best approximate students' information set, the survey had to take place after students had been notified of their score and had spent time researching colleges. Second, high school officials believed that the survey might distract some students from the high-stakes and time-sensitive application process, so we postponed the survey until after the college application deadline. Third, we could not wait very long after students had submitted their applications, because imperfect recall or self-rationalization upon receipt of admission outcomes might impede accurate measurement of their beliefs and preferences.

[11]The question asked students to guess the probability of meeting the admission cutoff of the college in question. Based on feedback from teachers and students in a pilot, admission cutoff is a very basic concept. The most natural way to elicit beliefs about admission probabiltiy was to ask students about their belief that their scores would meet the admission cutoffs.

3 Decision Problem

3.1 Setup and Mathematical Notations

To mathematically describe the decision problem in our setting, consider a student, Mei, who needs to list four colleges from a set of n colleges on her ROL. After she submits her ROL, along with m other students, the centralized admission system will process their ROLs using the Constrained Deferred Acceptance Algorithm (DAA). Then, Mei either will be assigned to one of the four colleges that she listed on her ROL, or will be rejected by all four colleges and end up with her outside option.

Following Agarwal and Somaini (2018), we assume that the number of students m is much larger than the number of colleges, n. In our empirical setting, n is much larger than four. Mei is playing an incomplete information game, where she does not know the ROLs submitted by other students and has to form beliefs about what the other lists could be.

As discussed in Section 2.1, as students have been notified of their scores and corresponding provincial rankings by the time of application, the admission outcomes solely depend on the cutoffs of the colleges that they apply for, which is unknown at the time of list submission. Consequently, Mei needs to form her beliefs about the distribution of cutoffs.

Based on beliefs about the distribution of cutoffs, Mei will assign probabilities of meeting the cutoffs to the n colleges that she is contemplating. Denote the unconditional probabilities (i.e., probability of meeting the cutoff) of the n colleges by p_1,p_2,...,p_n respectively. For the purpose of presentation and without loss of generality, assume that $p_1 < p_2 < ... < p_n$ (that is, colleges are ranked from the most competitive ones to the least). Denote the utility of admission to colleges by u_1, u_2, ..., u_n respectively. The utility of Mei's outside option is $\underline{u}$. For any single college j, the only two characteristics that Mei needs to care about are its admission utility u_j and unconditional admission probability p_j.

3.2 Estimate Probability p_j from Data

As discussed in the previous subsection, one of the two components of this decision problem is the unconditional probabilities p_j, for which we need to construct measures to approximate what students think. To that end, we proxy students' beliefs using admission cutoffs in the past, which are publicly available shortly after the end of each previous admission cycle[12].

The cutoffs in past years can reliably predict the current cutoffs. In Figure 2, we plot the cutoff in its converted form in a specific year (e.g., 2018) against the cutoff the previous year (e.g., 2017) for all the colleges that admitted Ningxia students during this time period. We find that the correlation between a cutoff and its past year counterpart is around 0.95; it would be even higher, except that a few outliers significantly drag the correlation down. The median of the distance between realized cutoffs and the college-level average during 2014-2018 is 0.097 of a standard deviation of the distribution of priority scores among science-track applicants, and 0.146

[12] See here for a well-known website that documents past admission cutoffs.

of a standard deviation of the distribution among humanity-track applicants. Importantly, the uncertainty is larger among less competitive colleges, as the scatter at the bottom left of each graph is more likely to be away from the 45-degree line.

To calculate probabilities, we assume that, for college j, in year t, the cutoff c_{jt} is normally distributed:

$$c_{jt} \sim N(\mu_j, \sigma(\mu_j))$$

where the value of μ_j is the average of admission cutoffs for college j during 2014-2018, which reflects the competitiveness of a college overall across years. The assumptions about the value of μ_j are motivated by the informativeness of past cutoffs. We compute the distribution of $c_{jt} - \mu_j$ in Figure A1a, and find that the distribution function is a bell-shaped function that is centered around zero, with reasonably thin tails, suggesting that it is possible to approximate the true distribution with a hybrid of normal distributions.

Based on the observation from Section 2 that the predictability is not homogeneous across colleges of different competitiveness, and to improve approximation accuracy as much as possible, we assume that the mean of $ln(\sigma_j)$ is a fourth-order polynomial of μ_j:

$$E[ln(\sigma_j)|\mu_j] = \beta_0 + \sum_{k=1}^{4} \beta_k \mu_j^k$$

. We estimate the model using maximum likelihood, for the science and humanity tracks, respectively. We then use the estimated $\hat{\beta}$ to predict σ_j:

$$\hat{\sigma_j} \equiv \hat{\beta}_0 + \sum_{k=1}^{4} \hat{\beta}_k \mu_j^k$$

With the estimation, the probability of meeting the cutoff of college j is:

$$\hat{p}_{ij} \equiv \Phi(\frac{s_i - \mu_j}{\hat{\sigma_j}})$$

where $\Phi(.)$ is the CDF of standard Gaussian.

We follow Kapor et al. (2020) to validate the estimates of admission probabilities. Specifically, we analyze individual level data on admission outcomes by running the following regression:

$$1(\text{Admitted to First Choices})_i = \alpha_1 + \beta_1 \hat{p}_{ij}$$

where subscript j represents students' first choices. The null hypothesis that the estimated admission probability is accurate implies that $\alpha_1 = 0$ and $\beta_1 = 1$.

The estimation results suggest that our estimates accurately reflect the actual unconditional admission probability. As shown in Columns (1) and (2) of Table B2, $\hat{\alpha_1} = -0.0043$ (SE=0.0024) and -0.0050 (SE=0.0040) for the science and humanity tracks, respectively, and $\hat{\beta}_1 = 1.0015$ (SE=0.0043) and 1.0117 (SE=0.0081) for science and humanity tracks, respectively. The p-values of the F-tests are 0.074 and 0.328, respectively, meaning that we fail to reject the null hypothesis.

To double-check that our estimates capture the observation that the uncertainty of cutoffs is larger for colleges of medium competitiveness, in Figure A1b we divide colleges into four groups of equal size according to their competitiveness and plot the kernel density estimation of the distribution of $c_{jt} - \mu_j$ for each group respectively. The takeaway from the graph is that our estimates do reflect the heterogeneity of volatility in admission cutoffs for colleges with different degrees of competitiveness, and that the normal distribution of different variances, as proposed above, can well approximate such empirical patterns.

3.3 Discussion of Utility u_j

Recovering college preferences is difficult, because u_j, potentially heterogeneous, is harder to estimate than unconditional probability p_j. Without reasonable estimates of u_j, however, it is impossible to infer whether or not a certain choice pattern is optimal (Agarwal and Somaini, 2019). A central issue that this paper will tackle is identifying sub-optimality in the presence of heterogeneous preferences.

However, we do know that much of the variation in u_j can be explained by the level of admission cutoffs. As admission cutoffs dictate outcomes, the level of the admission cutoff signals the extent of competitiveness of the college - the higher the cutoff is, the more competitive a college becomes. Since the cutoffs are very stable over time (Section 3.2), overall there is a consensus as to which colleges are more competitive and hence objectively more desirable. On the other hand, there is moderate variation in the level of cutoffs, suggesting two possibilities. First, echoing the hypothesis of this paper, some students may not be using the optimal decision rule, which results in decision heterogeneity and imperfect correlation. Alternatively, if everyone is making choices optimally, the imperfect correlation reflects idiosyncratic variation in preferences across years[13]. If the second possibility is true, the high correlation of cutoffs suggests that variation in horizontal preferences across years[14] is small relative to the vertical preferences[15].

3.4 The Optimal Rank-Order List

Given the beliefs about the unconditional admission probability of colleges, as well as the utility of admission for each college, playing Bayesian Nash Equilibrium in this context reduces to finding out the optimal portfolio for Mei. Specifically, suppose that the utilities and probabilities are (u_{j_1}, p_{j_1}), (u_{j_2}, p_{j_2}), (u_{j_3}, p_{j_3}), (u_{j_4}, p_{j_4}), respectively. Given Mei's ROL, she will be admitted to college 1 with probability p_{j_1}. Under Constrained DAA, she will be considered by college 2 only when college 1

[13]For example, if different types of students have different preferences such that they submit different lists, the admission cutoffs would vary as a function of the realized share of each student type.

[14]For the sake of convenience, we call orderings that defy the market consensus "horizontal preferences" (that is, $p_{j_1} > p_{j_2} \implies u_{j_1} \geq u_{j_2}$).

[15]For the sake of convenience, we call preference orderings that accord with market consensus (i.e., competitiveness) "vertical preferences" (that is, $p_{j_1} > p_{j_2} \implies u_{j_1} < u_{j_2}$).

has rejected her; thus, the probability of being admitted to college 2 is $(1 - p_{j_1})p_{j_2}$[16]. Similarly, the probabilities of being admitted to colleges 3 and 4 are $(1 - p_1 j_1)(1 - p_{j_2})p_{j_3}$ and $(1 - p_{j_1})(1 - p_{j_2})(1 - p_{j_3})p_{j_4}$, respectively. Therefore, her expected utility from the portfolio $\{j_1, j_2, j_3, j_4\}$ is:

$$EU([j_1, j_2, j_3, j_4]) \equiv p_{j_1}u_{j_1} + (1 - p_{j_1})p_{j_2}u_{j_2} + (1 - p_{j_1})(1 - p_{j_2})p_{j_3}u_{j_3} + (1 - p_{j_1}) \\ (1 - p_{j_2})(1 - p_{j_3})p_{j_4}u_{j_4} + (1 - p_{j_1})(1 - p_{j_2})(1 - p_{j_3})(1 - p_{j_4})\underline{u}$$

Considering both desirability and the chance of admission for a group of colleges at the same time substantially complicates the decision problem because the chance of admission at one college depends on the chance of admission at the colleges that the student has ranked above it. This interdependence implies that decisions should not be made by considering each program sequentially, viewed in isolation. Instead, Mei should consider admissions probabilities arising from a complete ROL, and thus optimal decision-making requires picking an optimal portfolio out of a large number[17], a computationally difficult task.

Chade and Smith (2006) proposes a computationally simple method that solves for this portfolio choice problem. Let dummy χ_j denote the event that a student's priority score meets the admission cutoff of college j. The utility of portfolio $\{j_1, j_2, j_3, j_4\}$ is defined as

$$PF(\{j_1, j_2, j_3, j_4\}) \equiv max\{\chi_{j_1}u_{j_1}, \chi_{j_2}u_{j_2}, \chi_{j_3}u_{j_3}, \chi_{j_4}u_{j_4}, \underline{u}\}$$

$E[PF(\{j_1, j_2, j_3, j_4\})] = EU(\{j_1, j_2, j_3, j_4\})$ when colleges are correctly ordered in the list according to their utilities, which is why we need a fifth step to rank all the colleges in the portfolio. Below is a summary of the five steps:

- **Step 1:** Choose college a such that $PF(\{a\})$ is maximized.
- **Step 2:** Given a, choose college b such that $PF(\{a, b\})$ is maximized.
- **Step 3:** Given a, b, choose college c such that $PF(\{a, b, c\})$ is maximized.
- **Step 4:** Given a, b, c, choose college d such that $PF(\{a, b, c, d\})$ is maximized.
- **Step 5:** order a, b, c, d such that the one with higher utilities is always ranked higher.

What are the implications of such procedures? We start with the standard scenario where there are no horizontal preferences (i.e., for any j_a and j_b, $p_{j_a} > p_{j_b}$, $u_{j_a} < u_{j_b}$), then address the more involved case where horizontal preferences exist. Suppose in both scenarios the right answer for Mei's best ROL is $[a^*, b^*, c^*, d^*]$.

[16] Here we are assuming that admission probability is independent because we believe that this is a reasonable approximation in our empirical setting. See Section 9.2 for a detailed discussion of why the assumption about independence is justified.

[17] To be precise, the number is $\binom{n}{4}$, which amounts to 4 billion in our context.

Vertical Preference Only In the absence of horizontal preferences, we know that $p_{a^*} < p_{b^*} < p_{c^*} < p_{d^*}$, a qualitative prediction that we can directly test using the administrative data alone. Intuitively, this happens because the optimal amount of risk taking for different positions is not the same. p_{d^*} needs to maximize $p_d u_d + (1 - p_d)\underline{u}$, whereas p_{a^*} needs to maximize $p_a u_a + (1 - p_a)EU([b, c, d])$. The fact that $EU([b, c, d]) > EU(\emptyset) = \underline{u}$ implies that Mei needs to worry less about the downside of pursuing competitive colleges (i.e., the utility that follows $(1 - p)$ in each expression) when contemplating the best a. Consequently, it is the colleges that the student ranks below the current choice, not the colleges ranked above it, that affect optimal choices. Backward induction, a decision rule unnatural to human cognition, becomes necessary - students need to think about a safe college first, and put it at the bottom of the list, and then gradually proceed, in reverse order, to the top of the list:

- (blank) $\rightarrow$ (blank) $\rightarrow$ (blank) $\rightarrow$ j_4 $\rightarrow$ (outside option)
- (blank) $\rightarrow$ (blank) $\rightarrow$ j_3 $\rightarrow$ j_4 $\rightarrow$ (outside option)
- (blank) $\rightarrow$ j_2 $\rightarrow$ j_3 $\rightarrow$ j_4 $\rightarrow$ (outside option)
- j_1 $\rightarrow$ j_2 $\rightarrow$ j_3 $\rightarrow$ j_4 $\rightarrow$ (outside option)

Horizontal Preferences In this case, we can no longer deduce $p_{a^*} < p_{b^*} < p_{c^*} < p_{d^*}$ from $u_{a^*} > u_{b^*} > u_{c^*} > u_{d^*}$, because, if any pair of colleges j_1 and j_2 that reflect Mei's horizontal preference are both present on the list, the more competitive one, which is less desirable in terms of Mei's subjective preference, would be ranked lower. Indeed, the issue of horizontal preferences is what much of this paper aims to address.

4 Reduced-Form Evidence on Admission Probabilities

Section 3.4 points to an empirical exercise that jointly tests the rational decision rule and perfect[18] vertical preferences. If this hypothesis is true, then:

1. The unconditional admission probability in the first choice is quite low.
2. For each ROL, admission probability of any lower-ranked colleges should be higher than that of higher-ranked colleges.

The discussion in Section 3.3 demonstrates that the impact of idiosyncratic preferences might be limited. Therefore, if we find systematic violation in these two measures, this could be interpreted as a rejection of the joint hypothesis that tentatively favors the existence of suboptimal choices.

Section 4.1 aims to conduct these empirical exercises, where we look at the distribution of unconditional probabilities for top choices as well as the cases where unconditional probabilities

[18]For any college j_1 and j_2 such that $p_{j_1} > p_{j_2}$, $u_{j_1} < u_{j_2}$. Note that this word is agnostic about preference intensity - that is, how much higher u_{j_2} is compared to u_{j_1}.

are not ranked in order. To establish a benchmark for comparison, in order understand whether empirical evidence favors the hypothesis, in Section 4.2 we simulate students' college choices by assuming that college preferences are governed by simple utility specifications. In Section 4.3, we examine whether socioeconomically disadvantaged students deviate more or less from the prediction of the rational benchmark.

An important feature of our setting is that some students who barely meet the minimum requirement of first-tier colleges have limited options because of their low priority score. Therefore, mechanically they are much more risk-taking than those whose priority score is high. Figure A2a presents the bin-scatter plot of the mean probability for the four choices conditional on the quantile of priority score. We can see from the figure that, for the bottom 5% in terms of priority score, even the mean probability of the fourth choice is less than 20%, as these students really don't have any safe choices. The mean probability for all the four choices rises simultaneously, with this trend stopping when the priority score quantile is around 40%. We therefore focus on the top 60% in this and subsequent sections.

4.1 Summary Statistics on Admission Probability

Distribution of Admission Probability for the Four Choices Figure 3b presents the box plot of unconditional admission probability that we construct as described in Section 3.2, for each choice on the lists. The median of the probability, as indicated by the white dashed line, is 47.0%, 83.7%, 95.9%, 99.8%, respectively, consistent with the prediction from the rational benchmark that students should be pursuing more risks for top choices. However, the extent of risk taking is limited for non-first choices, as suggested by the medians for non-first choices. This implies that, if students are indeed following the rational decision rule, they should be quite averse to risks.

The heterogeneity of unconditional probability for the first choice is massive, with the 75th percentile being 88.5%, suggesting that a substantial portion of students are not taking much risk at all, even for their top choices. On the other hand, the heterogeneity of probability is minimal for the fourth choices, where the 25th percentile is 95.4%, suggesting that the vast majority of students are taking little risk for their bottom choices, which makes sense in a high-stakes environment. However, summary statistics in probability alone are insufficient, as we need to calibrate students' choices to map the level of probability to familiar measures in risk preferences.

Share of "Risk-Taking Reversal" The rational decision rule and vertical preferences jointly predict that the probability should be lower for the higher-ranked choices. To quantify students' strategy in this dimension, we construct risk-taking reversal, namely, a "flip" of the probability of colleges, to quantify the violation of benchmark prediction. As our goal is to capture risk-taking reversal anywhere on the ROL, we consider the following statistics:

$$R \equiv max_{j>i}\{p_i - p_j\}$$

In the expression of R, we take the maximum for the probability gap between any pair of choices to capture the most serious risk-taking reversals on the ROLs. We plot the results in Figure 3c, and find that more than 55.8% of the ROLs exhibit risk-taking reversals ($R > 0\%$). Further limiting our scope to the case of "serious" reversals, where an ROL is counted only when R exceed a certain positive threshold ($R\% > 25\%$,$R\% > 50\%$,$R\% > 75\%$), the share mechanically decreases but still remains non-negligible. For example, the share is 24.6% when the restriction is $R\% > 25\%$.

In summary, the data suggests that a substantial proportion of students are quite cautious even for their first choices, and many of them exhibit "risk-taking reversals" on their lists. This clearly rejects the joint hypothesis that students have perfect vertical preferences and are following the rational benchmark. As Section 3.3 has demonstrated that horizontal preferences tend to have a limited impact on college choices, it is likely that some students are not following the rational benchmark. Quantitatively, however, we do not know whether the existence of such students is significant enough to alter the choice patterns.

4.2 Calibrating Rational Benchmark

We have documented the stability of cutoffs (Section 3.2), substantial heterogeneity in the first choices, and presence of reversals (Section 4.1). To quantify whether these observations could be rationalized by risk preferences (curvature over competitiveness) that alter students' intensity of vertical preferences, or random idiosyncratic shocks to students' preferences as a proxy for horizontal preferences, we calibrate a very simple model where the utility of college j for student i is:

$$u_{ij} = \frac{(c_j - m)^{1-\rho} - 1}{1 - \rho} + \epsilon_{ij}$$

where c_j is the normalized competitiveness of college j as measured by average cutoffs during 2014-2018. $m \equiv min_x\{c_x\}$ is the lowest value of the college in this measure, so that $(c_j - m)$ is guaranteed to be positive, with the standard deviation being 1. ρ represents the curvature of preferences over the competitiveness of a college. $\epsilon_{ij} \sim N(0, \sigma^2)$ is the idiosyncratic preference shock over colleges. This simple model captures the core components of preferences: vertical preferences, as controlled by the curvature ρ, and idiosyncratic horizontal preferences, controlled by the level of σ. Lower ρ and σ imply that a larger share of variation in preferences is explained by vertical preferences.

We focus on three specific statistics for the calibration: correlation of cutoffs in 2017 and 2018, mean probability of first choices, and the share of reversals. Intuitively, we would expect that higher curvature, i.e., larger ρ, would lead to more cautiousness for the first choices, and that more horizontal preferences, i.e., larger σ, would lead to lower correlation in cutoffs and more risk-taking reversals. As shown in Figure 4, holding one of the parameters constant, the effect of the other accords with the intuition.

However, the tension between these statistics highlights the impossibility of rationalizing these three important observations at the same time. Reversals can only be rationalized by introducing horizontal preferences, i.e., increasing σ. A larger idiosyncratic shock, however, also decreases the stability of cutoffs because competitiveness contributes to a smaller share of utility in that case. As shown in Figures 4b and 4c, creating a substantial share of reversal necessitates significantly lower correlation in cutoffs. In other words, the stability of cutoffs bounds the level of idiosyncratic shock allowed in the specification. Quantitatively, if we focus on the case of linear utility $\rho = 0$, σ that does not exceed 0.1 would match the level of correlation that we see in the data, suggesting that, from an "R-squared perspective," the variation in vertical preferences needs to contribute to $\frac{1^2}{1^2+0.1^2} \approx 99\%$ of the total variation in utility across years.

Moreover, as shown in Figure 4a, the curvature that rationalizes the cautiousness for the first choices is quantitatively implausible and needs additional restrictions in horizontal preferences. When competitive colleges are highly sought after, as in our context, the utility differences between the top college and the median, Δu_{tm}, should be no smaller than the utility difference between the median and the bottom, Δu_{mb}. In this case, while the mean of probability from the data is 49.7%, the simulated mean when $\Delta u_{tm} \geq \Delta u_{mb}$ ranges from 22.2% to 31.4% regardless of the level of idiosyncratic shock, roughly 20% less than what the data reveals. This level of cautiousness is achieved only when students are almost indifferent between the median and the bottom ($\Delta u_{tm} < 10^{-7}\Delta u_{mb}$), under the assumption that there are no horizontal preferences at all.

Surprisingly, when just a little idiosyncratic taste heterogeneity is introduced, the cautiousness wanes and the simulated mean of probability drops to less than 45% no matter how small Δu_{tm} is. This happens because idiosyncratic preferences work as a "double-edged sword" when it comes to cautiousness. When a college is particularly attractive, students select it as the first choice even if its probability is quite high, reflecting an increase in cautiousness. However, students will also be selecting it as the first choice when its probability is lower. These two effects counteract each other, and in our data the latter effect dominates when the curvature is extreme.

This highly stylized model certainly omits horizontal preferences that are stable and persist across years. However, it highlights the difficulty of simultaneously rationalizing our observations about the stability of cutoffs, cautiousness in the first choices, and reversals using idiosyncratically heterogeneous preferences. In sum, the rejection indicates that either more complicated horizontal preferences or alternative behavioral considerations are needed to explain these three empirical patterns at the same time.

4.3 Socioeconomically Disadvantaged Depart Further From Rational Benchmark

A large literature documents that socioeconomically disadvantaged students are particularly bad at strategizing in centralized systems (Lucas and Mbiti, 2012; Ajayi, 2013; De Haan et al., 2015; Shorrer and Sóvágó, 2018; Kapor et al., 2020). To examine whether demographics are important in our context, we choose average educational attainment at township level[19] to approximate so-

[19] Roughly speaking, township is equivalent to zip code in the US.

cioeconomic status[20]. As we also have access to students' home addresses in 2015, 2017, and 2018, we match the township level educational attainment data to individual students in the administrative dataset, and plot the distribution of this measure in Figure A3. The figure demonstrates sizable heterogeneity in terms of educational attainment across different townships in Ningxia (25th percentile: 7.80 years; median: 8.73 years; 75th percentile: 9.88 years). In the following exercise, we split students into four groups according to their SES, and focus our attention on the most advantaged quartile (edu $>$ 9.88 years) and the most disadvantaged quartile (edu $<$ 7.80 years). Importantly, as Ningxia accounts for only about 0.7% of China's area, and all but two first-tier colleges are located far outside the province, the difference in township should not significantly alter applicants' preferences over physical distance.

In Figure 5a, we compute the statistics in Section 4.1 for the most advantaged and the most disadvantaged quartile, respectively. All the statistics have been reweighted to take into account any differences in priority score, so the patterns cannot be explained by students facing different choice sets due to differential priority scores.

The mean probability of the first choices among the socioeconomically disadvantaged students (53.8%) is less than their advantaged counterparts (45.7%), after priority scores have been controlled by reweighting. However, for their fourth choices, the probability among the disadvantaged (91.3%) is slightly less than their advantaged counterparts (92.9%) [21].

Do the differences hold statistically? We use the following specification to quantify the difference:

$$\text{Outcome} = \beta\text{Disadv} + f(\text{Priority Score}) + \text{Disadv} * g(\text{Priority Score}) + \text{other controls} \tag{1}$$

Where Disadv indicates whether students are from a lower SES group, f(Priority Score) and g(Priority Score) represents a fourth-order polynomial of priority score[22]. The main effect of Disadv, β, is the overall outcome gap between students of different SES groups after priority score has been fully controlled, as well as the heterogeneity in whether the outcome gap changes with the priority score.

As shown in Table 1, the results confirm our visual perception regarding the first and fourth choices. After introducing the full set of controls as listed in Panel C, the mean probability of the first choices among the disadvantaged is 6.74% (SE 0.40%) more than their advantaged counterparts (Column 1), and the mean probability of the fourth choices among the disadvantaged is 2.24% (SE 0.40%) less than their advantaged counterparts (Column 2). As a result, the gap between the advantaged and the disadvantaged with regard to fourth-first choice probability differences amounts

[20] We obtain statistics on average years of education among adults between 40-65 at township level from the China Census in 2010.

[21] Please refer to Figure A2b for a complete breakdown of admission probability by priority score, across students in the most advantaged and most disadvantaged quartiles.

[22] Each of the terms in the polynomial has been demeaned so that the main effect β is the predicted Adv-Disadv Gap at average level of priority score.

to 8.98% (SE 0.78%). Panel B demonstrates that the signs of the Adv-Disadv gaps remain the same as the main effect in corresponding columns over the whole range of priority scores, though the effect is insignificant among those with the highest priority scores.

Figure 5b plots the share of reversals for the most advantaged and disadvantaged quartiles, respectively. After differences in priority scores have been adjusted, the gap in the share of reversals between disadvantaged and advantaged remains about the same, and is robust to the threshold. When the threshold $X\%$ is 25%, for example, the share of reversals among the advantaged is 19.7%, and the weighted share of reversals among the disadvantaged is 26.1%, roughly 40% higher than the advantaged.

In Table 2 we consider the following regression:

$$1(R > X\%) = \text{Disadv} + f(\text{Priority Score}) + \text{Disadv} * g(\text{Priority Score}) + \text{other controls} \qquad (2)$$

where we vary the threshold X so that it equals 0, 25, 50, or 75% in Columns 1, 2, 3, and 4, respectively. As reported in Panel A, the qualitative results, after priority score has been fully controlled for, are consistent with the graphical observation, with the gap in the share of reversal remaining at about 5%. As shown in Column (2), for example, after controls listed in Panel C are included, the estimated share of reversal (threshold $R > 25\%$) is 6.64% (SE 0.80%) higher among the most disadvantaged. As presented in Panel B, the gap between the disadvantaged and their counterparts persists at different levels of priority score, though sometimes insignificantly so among those with the highest priority scores.

We also restrict our simulation exercise in Section 4.2 to the most disadvantaged quartile. Figure A4 presents the main results. The takeaway is that the disadvantaged are more cautious in their first choices (unadjusted mean: 50.9%) and exhibit even more risk-taking reversals (unadjusted mean: 28.0%), such that the simple utility model has a harder time rationalizing their choice patterns. For example, the share of reversals among the disadvantaged never exceeds 23.9% even with substantial idiosyncratic shock ($\sigma = 1$) and ($\Delta u_{tm} \approx 0$).

To examine whether pursuing different strategies systematically affects admission outcomes among the disadvantaged, we run the following two groups of regressions:

$$\text{Selectivity of Admitting College} = 1(\text{SES Quartile}) + f(\text{Priority Score}) + \mathit{Controls}$$

where we measure the selectivity of the admitting college by calculating the average admission cutoffs for the colleges during 2014-2018. We report the regression results in Table B3. The results suggest that socioeconomically disadvantaged students on average end up in less selective colleges compared to their advantaged counterparts, even after other demographic variables that are correlated with socioeconomic status have been included. In one of our specifications, the most disadvantaged quartile (1st Quartile) on average end up in colleges whose selectivity is

0.1046 (SE=0.0052) of a standard deviation worse compared to the most advantaged quartile (4th Quartile).

5 The Model of Directed Cognition

5.1 Formulation of DC Decision Rule

The rational benchmark in Section 3.4 proposes that the optimal portfolio can be reached by decomposing the problem into four different discrete choice problems. The correct decomposition of this problem requires student applicants to invoke the logic of backward induction, because it is the colleges that they list below a given rank, not those above that rank, that affect the optimal choice for the given rank. Literature in experimental economics, however, has established that, even in simplified settings, subjects have trouble grasping the concept of backward induction (Camerer et al., 1993; Johnson et al., 2002). Moreover, laboratory evidence also suggests that decision makers lack the ability to cope with uncertainty in simple decisions (Martínez-Marquina et al., 2019), a skill that is necessary to assess the distribution of utility for the lower-ranked choices.

In this section, we continue to use the setting in Section 3, and introduce an alternative boundedly rational decision rule that is inspired by the Model of Directed Cognition in Gabaix et al. (2006) (henceforth, DC). We believe that this decision rule explains students' suboptimal strategies when they cannot apply the optimal strategy as prescribed in Chade and Smith (2006). Consider another college applicant, Hua. The rule prescribes that, instead of tracking all the information and acting optimally upon it, Hua, due to cognitive limitations typical in students his age, myopically focuses on the spot where he is actively contemplating which college to fill in, and neglects the impact of that choice on the rest of the list. He fills out his ROL in a natural order, from the first choice to the fourth choice. As a result, in each step Hua is making choices for essentially the *same* decision problem. That is, he maximizes the expected utility of a portfolio that consists of his current choice and the (subjective) outside option:

- j_1 → (blank) → (blank) → (blank) → (Outside Option)
- j_1 → j_2 → (blank) → (blank) → (Outside Option)
- j_1 → j_2 → j_3 → (blank) → (Outside Option)
- j_1 → j_2 → j_3 → j_4 → (Outside Option)

This decision rule requires less cognitive capacity for two reasons. First, as Hua proceeds in a natural order by considering the first choices before the rest of the ROL, the logic of backward induction is no longer necessary. Second, as Hua is making choices for each spot in isolation, the source of uncertainty is reduced and he only needs to consider at most two states: admission to a first-tier school, or rejection by all four of his choices and getting the utility of the outside option.

Such myopic decision making comes at a cost. Compared to the rational benchmark, the DC rule induces Hua to act cautiously. Take the first choice on his list, for example; when he adopts the DC rule, his perceived utility from backups is $\underline{u}$, far below the actual expected utility from the colleges that follow it $EU([b, c, d])$. This difference is most pronounced at the first choice of the ROL because that is where the difference between *actual* and *perceived* expected utility from the options ranked below is maximal. In contrast, a DC type's risk-taking behavior would not differ substantially from its rational counterparts for the fourth choices of ROL because that is where the *actual* and *perceived* expected utility coincide.

5.2 Predictions: Cautiousness and Reversal under Vertical Preferences

Compared to the rational decision rule, the DC decision rule yields substantially different predictions about the patterns of risk taking. To mathematically describe the differences between the rational benchmark and the DC rule, consider Hua, who is considering which of two colleges, a or b, to list as his rth choice. College a is more desirable and riskier than b, thus $p_a < p_b$. Hua needs to compare $U_a \equiv p_a u_a + (1-p_a)U_r$, the expected utility of choosing a, to $U_b \equiv p_b u_b + (1-p_b)U_r$, where U_r is the perceived expected utility of a portfolio that consists of all the choices that are listed below rth choice, and the outside option. We call this combination "backup list below the rth choice". Define the propensity to take risks as a function of the position of choice r and the decision rule (Optimal or DC)

$$f(r, \text{decision rule}) \equiv U_a - U_b = p_a u_a - p_b u_b + (p_b - p_a)U_r$$

Here, the greater $U_a - U_b$ is, the more appealing it is to take risks and choose a.

Similar to the discussion in Section 3.4, we first focus on the case where the order of utility follows "market consensus" (vertical preferences), i.e., $u_a > u_b$. Later we will discuss the case where Hua has his own horizontal preferences over these two colleges, i.e. $u_a \leq u_b$.

We have:

Prediction 1 (Cautiousness at Top) *Compared to a DC type, a rational type takes substantially more risks in listing their first choice. Moreover, the gap between risk taking at the first and the fourth choices is much smaller for the DC type. Mathematically,* $f(1, DC) < f(1, Rational)$ *and* $f(1, DC) - f(4, DC) < f(1, Rational) - f(4, Rational)$

.

This prediction holds because Mei can correctly calculate the expected value as $U_1 > U_2 > U_3 > U_4 = u_0$, whereas Hua ignores everything below except the outside option and thus $U_1 = U_2 = U_3 = U_4 = u_0$. As $U_a - U_b$ is increasing in the expected utility of backup list, Mei is more inclined than Hua to take a risk in her first choice. The propensity to take risks does not vary for Hua across the list because his perceived backup list is always the outside option, regardless of which choice on the list he is contemplating. By contrast, Mei's cautiousness is as great as Hua's for her fourth choice, but much less than Hua's for her first choice. Thus the presence of the DC type will increase the mean probability for the first choices.

The DC decision rule also has implications for the order of admission probability for the listed colleges:

Prediction 2 (Risk-Taking Reversals) *Under perfect vertical preferences, the ROL of a rational type always features decreasing utility and increasing admission probability (that is, $u_a > u_b > u_c > u_d$, $p_a < p_b < p_c < p_d$). A DC type, in contrast, may exhibit "risk-taking reversal" by putting a riskier college in a lower-ranked position, leading to dominated choices.*

This prediction holds because, for any given list, we have $f(i, \text{Rational}) > f(j, \text{Rational})$ for any $i > j$ for Mei. Thus, the optimum for the rth choice is more risky for her. For Hua, however, $f(i, \text{Rational}) = f(j, \text{Rational})$ because only the outside option is regarded as his backup list. Thus the presence of the DC type will increase the share of reversals.

5.3 Prediction: Framing Effect under Arbitrary Preferences

This subsection presents a prediction that tells apart the two decision rules without any assumptions on college preferences. The prediction is testable with carefully designed survey questions, as detailed in Section 6.1 and 6.2. To understand this prediction, let's consider what leads to a DC type's suboptimal strategy. The correct utility of choosing a for the rth choice, as discussed previously, is

$$U_a \equiv p_a u_a + (1 - p_a) U_r$$

This expression implies that, with probability p_a, the utility is u_a; with probability $(1 - p_a)$, the utility is U_r. A DC type's trouble is that when the question is presented in a ROL, they fail to calculate U_r and instead treat it as u_0. When this is not presented in the form of a ROL question, but in the form of the mathematically equivalent lottery choice $(p_a, u_a; (1 - p_a), U_r)$, the payoff in the event of rejection has been calculated and presented clearly so that the DC type cannot distort it. Effectively, choices present in the form of lottery can "de-bias" by bring the utility of backup choices to their attention so that their cognition is no longer directed to a single spot. Let U'_r denote the perceived expected utility of the portfolio that follows the rth choice. We have the following prediction:

Prediction 3 (Framing Effect) *Suppose all the possible portfolios that are framed as ROL have been correctly transformed into their lottery representation. In that case, the rational type will behave consistently across ROL and lottery questions. By contrast, a DC type will be more risk-taking in the lottery questions.*

This prediction holds because, for the rational type, $U'_r = U_r$, regardless of whether the problem is presented in lottery representation or ROL representation. For the DC type, $U'_r = U_r$ if it is presented in lottery representation, but $U'_r = u_0 < U_r$ if it is presented in the ROL representation.

5.4 Prediction: Position Movement in ROL under Arbitrary Preferences

This subsection presents another prediction that tells apart the two decision rules without any assumptions on college preferences. The prediction yields predictions that can be tested non-parametrically using the administrative dataset alone, thanks to the unique feature of our setting that each student's priority score is determined by exam performance and is not affected by the student's choice of colleges. The data analysis concerning this prediction is presented in Sections 7.1 and 7.2.

We characterize the choice pattern of any single college, A, as priority score changes. For an arbitrary preference profile u (a utility vector that represents preferences over *all* colleges), if A appears on the list given a specific priority score s, where would A be if s were higher?

To understand the result intuitively, consider under rational decision rule and preference u, which means that A is a particularly attractive college and will appear on the list for some s. If s is too low, such that being admitted to A is impossible, A will be omitted because listing it wastes a spot. However, as s becomes higher, A appears on the list as soon as the admission probability is high enough. As s continues to move higher, more colleges become possible options for u. Consequently, A will remain in the same place if none of the newly possible options are better than A, and will move down the list if any newly possible option is better than A and has a favorable chance. Figure 7a presents an example in which the position of a college evolves as priority score changes.

To mathematically describe the result, define function $\mathcal{R}$ that maps preference profile u, priority score s, college A to its position κ on the list under the rational decision rule:

$$\mathcal{R} : (u, A, s) \mapsto \kappa$$

where κ is 0 if A is omitted from the list. κ **takes the value of 4, 3, 2, 1 if A is the first, second, third, and fourth choice, respectively.** The following theorem characterize $\mathcal{R}$:

Theorem 1 *Suppose the cumulative distribution of cutoffs is continuous and log-concave*[23]*. For any preferences u, college A, and s_0 such that $\mathcal{R}(u, A, s_0) \geq 1$, if $s > s_0$, then $\mathcal{R}(u, A, s) \leq \mathcal{R}(u, A, s_0)$. Moreover, for any $\kappa \geq 1$, the set $\mathcal{C}^{RN}_{(u,A,\kappa)} = \{s|\mathcal{R}(u, A, s) = \kappa\}$ is connected.*

Similarly, define function $\mathcal{D}$ that maps preference profile u, priority score s, college A to its position κ on the list under DC decision rule:

$$\mathcal{D} : (u, A, s) \mapsto \kappa$$

$\mathcal{D}$ is different from $\mathcal{R}$ in that a particular college need not move down the list as s increases. College A is not listed when s is too low for similar reasons. As s increases, however, it first appears at the fourth choice once its probability is just high enough to exceed the previously fourth

[23]Log-concave distributions include normal distribution, uniform distribution, exponential distribution, logistic distribution, extreme value distribution, Pareto distribution, etc.

choice, which is the least appealing one on the list in terms of $p_A u_A$[24]. If u_A is higher than other listed colleges (despite lower probability, which results in lower $p_A u_A$ overall), it may move ***up*** as s increases, because p_A affects how A is ranked under the DC rule. It starts to move down gradually if s is so high that better colleges are within the reach. Figure 7b presents an example where the position of a college evolves as priority score changes. Compared to Figure 7a, it becomes apparent that the "climbing up to the top" movement to the left of the peak in Figure 7b distinguishes DC from the rational rule. The following theorem mathematically characterizes $\mathcal{D}$:

Theorem 2 *Suppose the cumulative distribution of cutoffs is continuous and log-concave, and the tie of expected utility happens only between pairs of colleges. For any preferences u and college A, if there exist $\underline{s}$ and $\bar{s}$ such that $\mathcal{D}(u, A, \underline{s}) = 0$ and $\mathcal{D}(u, A, \bar{s}) = \kappa$, then for any integer $\xi \in [1, \kappa]$, there exist $s \in [\underline{s}, \bar{s}]$ such that $\mathcal{D}(u, A, \bar{s}) = \xi$.*

The derivations of both theorems are detailed in Appendix C. Note that the assumption about the tie of expected utility is not essential, because, if we assume that students choose randomly if more than two colleges tie, similar results emerge as well. Theorem 1 and 2 together imply the following prediction:

Prediction 4 (Upward Movement) *If the priority score increases, for any preferences, previously listed colleges will only move downward on the list under rational decision rule, but may move upward under the DC decision rule.*

6 Support for Directed Cognition: Testing Framing Effect

The predictions concerning cautiousness and risk-taking reversals in Section 5.2 can explain the findings that we discuss in Section 4. However, these predictions are made under the assumption of perfect vertical preferences. One prediction that differentiates the DC rule from its rational counterparts under arbitrary preferences, as discussed in Section 5.3, involves "framing effects". This section introduces details about how the survey was conducted, as well as the data analysis that relates to the testing of this prediction.

6.1 Details of Survey Questions

The first and third group of questions asked students to take the amount of risks they prefer in each hypothetical situation. They need to choose one from College X or College Y, whose payoffs in the event of admission and unconditional admission probability have been specified in Panels A2 and C2 of Table B4 respectively, and fill in the first spot of the ROL. For each MPL, there are seven questions in total, as presented in the table, where the payoff of X is held constant (admission probability is 50%, get 25 CNY if "admitted" in this scenario), and the payoff of Y is in increasing order (admission probability is 25%, get 30, 35, 40, 45, 50, 55, 60 CNY if "admitted" in

[24] For the sake of convenience, the outside option in this subsection is assumed to be 0.

this scenario). The second, third, and fourth spots of the ROL have been pinned down, as shown in Panel A1 and C1 of Table B4, where one of them will definitely "admit" the student applicant if she is not "admitted" to the first spot. The rules of admission for both groups of questions are exactly the same as the real admission procedures, with the only difference being that the payoff of being "admitted" to lower ranked colleges in Question Group 1 is 20 Chinese Yuan (CNY), whereas the amount in Question Group 3 is merely 5 CNY. These binary choice problems feature the core tradeoff in our setting: if students wish to take more risks and choose a more desirable college for their first choices, they must face a greater risk of being admitted to backup choices, whose payoffs are considerably lower.

The second group of questions is mathematically equivalent to the first one, but is asked in the form of its lottery representation. This group has seven questions as well. Lottery X, whose payoff structure is (25 CNY, 50%; 20 CNY, 50%) delivers the same distribution of payoffs as College X in Question Group 1, and is held across questions. The payoff structure of Lottery Y is (30 CNY, 25%; 20 CNY, 75%), (35 CNY, 25%; 20 CNY, 75%), ... (60 CNY, 25%; 20 CNY, 75%), respectively, mathematically equivalent to the payoff of College Y in Question Group 1. In terms of framing, however, Question Group 2 differs from Question Group 1 in that the probability and payoff of the state when rejection of the first choice occurs is included in the choices, as shown in Panel B of Table B4. Because the DC Type in our setting choose their first college in isolation, they ignore the payoff of lower-ranked colleges, and thus fail to translate the ROL problem to its correct lottery representation. The inclusion of downside payment in the lottery precisely mutes the mistake from the DC decision rule.

How do responses to these survey questions map onto attitudes of risk taking? Since the payoff of College/Choice X is held constant, whereas that of College Y is increasing, a coherent response could switch from X to Y at most once. Because Y is the riskier option, the earlier the switching points, the more risk-taking the students.

We took several measures to minimize misunderstanding and maximize the efficacy of survey responses. First, students were directed to correctly read through our explanations about the questions and complete comprehension checks before answering these questions. Second, the college names in the table were customized according to survey takers' CEE score: both College X and Y on their screen were real Chinese colleges that are within their reach, with Y being more competitive; backup spots are filled with local colleges, which are generally considered to be worse given the under-developed status of Ningxia. Third, to minimize the possibility of personal preferences interfering with choices in ROL questions, we framed the questions as students themselves making choices for their hypothetical friends who share their risk preferences.

6.2 Analysis of Survey Data

Prediction 3 states that DC type students appear to take less risk in ROL questions compared to their (mathematically equivalent) lottery representations. We start our analysis by plotting the joint distribution of students' responses to Question Groups 1 and 2 in Figure 6. About 43% of the data points are located around the diagonal line (blue blocks), which represent students who take similar levels of risk across these two questions, i.e., exhibit consistent risk preferences across equivalent questions that are merely framed differently. This is consistent with the rational decision rule. Meanwhile, 51.9% of data points are located at the top right (red blocks), indicating that some students are taking substantially less risk in the ROL question compared to its lottery equivalent, consistent with the DC decision rule.

Are there more DC type students among disadvantaged students? This is suggested by Section 4.3, where socioeconomically disadvantaged students depart farther from the prediction of the rational benchmark. In order to answer this question, we need to quantify the extent of inconsistency. The two groups of questions enable us to identify the "indifference point" for the upside payment above which the riskier option is more desirable. Let R denote the amount such that the student is indifferent between College X (25 CNY, 50%) and College Y (R CNY, 25 %) and between Lottery X (25 CNY, 50%; 20 CNY, 50%) and Lottery Y (R CNY, 25%; 20 CNY, 75%). We run the following regression to test whether disadvantaged groups, on average, answer these two groups of questions more inconsistently:

$$|R - Y| = \beta \text{SES Index} + \text{Controls}$$

where SES Index is parents' average years of education, as elicited from the data. We report the results in Table 3, Panel A. Consistent with Figure 6, the inconsistency of response across the two groups of questions is substantial, with the mean of the inconsistency measure being 11.618. Moreover, the inconsistency is more pronounced among the socioeconomically disadvantaged students, and the relationship is robust across different specifications. With all available controls included, a 1 SD increase in SES Index is associated with a 1.337 (SE=0.360) decrease in the inconsistency measure.

To estimate the share of the DC type in the survey sample, we model survey takers' risk-taking behavior by assuming that students have constant relative risk aversion (CRRA) preferences:

$$u = \frac{(c+B)^{1-\rho} - 1}{1-\rho}$$

where B is background consumption, which we set to be 10 CNY[25]. We allow ρ to vary on Edu, parents' average years of education, and CEE, the quantile of Priority Score:

$$\rho = \alpha_0 + \alpha_1 \text{SES} + \alpha_2 \text{Score} + \epsilon_\rho$$

where $\epsilon_\rho \sim N(0, \sigma_\rho^2)$. Following Von Gaudecker et al. (2011), we model the noise in decision making in terms of its impact on perceived certainty equivalents of a risky choice. Decision noise $\epsilon \sim N(0, \sigma^2)$ is independent across different questions and will govern the choice: for example, if an individual prefers College $X \equiv (25, 50\%; 20, 50\%)$ to $Y(m) \equiv (m, 25\%; 20, 75\%)$ when $m = 30$,

[25] Roughly the value of one meal in China.

but switches to $Y(m)$ when $m \geq 35$, this means that:

$$CE(X, \rho) + \epsilon \geq CE(Y(30, \rho))$$

and

$$CE(X, \rho) + \epsilon < CE(Y(35, \rho))$$

where $CE(L, \rho)$ is the certainty equivalent of lottery L when the CRRA coefficient is ρ.

Regardless of type, students with the same ρ in our model will produce the same distribution of choices in the standard lottery question (Question Group 2). However, in Question Groups 1 and 3, different types of individuals behave differently despite having the same risk preferences. Denote the lottery presentation of college X and Y from Question Groups 1 and 3 by $X(b) \equiv (25, 50\%; b, 50\%)$ and $Y(m, b) \equiv (m, 25\%; b, 75\%)$, respectively, where m is the payoff of first choices and b is the payoff of backup colleges. While the rational type will correctly translate the ROL question to its lottery representation, a DC type processes the ROL questions differently:

$$CE_{\text{DC Type}}(X(b)) = CE(X(0))$$

and

$$CE_{\text{DC Type}}(Y(m, b)) = CE(Y(m, 0))$$

Thus, a DC type distorts the downside of the lottery to a lower level, and appears to be more cautious.

We additionally consider a third type that is established in the literature for some of our specifications: the ”sincere type” (Pathak and Sönmez, 2008; Calsamiglia et al., 2020). To put it into our context, the sincere type predicts that, in the ROL presentation, students will always prefer colleges with the highest payoffs, ignoring the probability of admission. In terms of certainty equivalents:

$$CE_{\text{Sincere Type}}(X(b)) = 25$$

and

$$CE_{\text{Sincere Type}}(Y(m, b)) = m$$

While we do not find much evidence supporting the presence of this type in administrative data, as very few students consistently choose overly competitive colleges throughout the whole ROL, this type can be well identified in our survey.

We further assume in this mixture model that the share of the DC type varies with socioeconomic status:

$$Prob(\text{DC Type}|\text{Edu}, \text{Score}) = \frac{exp(\beta_0 + \beta_1\text{SES} + \beta_2\text{Score})}{exp(\beta_0 + \beta_1\text{SES} + \beta_2\text{Score}) + 1}$$

We estimate this mixture model and present the main results in Table 3, Panel B. Including

DC Type in the estimation (Column 2) substantially improves the fit (Log Likelihood = -5770.302, 9 parameters) compared to a model in Column 1 that only allows for the rational type (Log Likelihood = -6271.152, 6 parameters). The improvement is substantial, such that the Bayesian Information Criterion also favors the mixture model (BIC metrics = 11605.88) over the single-type rational model (BIC metrics = 12585.82). The impact of including the sincere type (Column 3), while it also improves the fit, is limited compared to the DC Type (Log Likelihood = -5733.326, 10 parameters). Consistent with this observation, the estimated share of the DC type is 48.7% in our preferred specification, whereas the share of the sincere type is about 3% (Column 3). The marginal effect of socioeconomic status is also statistically significant, where a 1 SD increase in the SES Index is associated with a 7.3% (SE=1.7%) decrease in the share of the DC type. Lastly, heterogeneity of risk preferences plays little role in fitting the data, and overall the risk aversion among the survey takers is moderate (mean of CRRA coefficient 0.079, SE=0.013).

7 Support for Directed Cognition: Testing Upward Movement

In this section, we use administrative data to test another prediction that differentiates the DC rule from its rational counterparts under arbitrary preferences. We called this "upward movement." Different from Section 4, we focus on analyzing how choice patterns change as priority scores vary. As discussed in Section 5.4, the prediction of the DC rule regarding change in the choice patterns systematically differs from its rational counterparts.

7.1 Data Construction

This subsection presents how we reorganize the administrative data to test the theoretical predictions discussed in Section 5.4. To create the best environment for this test, we need to address two issues. First, the right counterfactual needs variation in priority scores that are orthogonal to distribution of preferences. Second, while Prediction 4 draws a clear line between the pattern of the DC rule and its rational counterpart for any given preference u, in real data the heterogeneity of u prevents us from observing any such clear patterns. Fortunately, both issues can addressed, as discussed below.

(I) Variations in priority score that are orthogonal to preferences As students cannot perfectly control exam performance, conceptually, there will be random variation of priority scores among students with the same academic ability. This conceptual variation is hard to quantify, but anecdotal evidence suggests that it is typical to have a CEE score 30-35 points away from performance in mock exams.[26] Moreover, even conditional on the same exam performance, the same academic ability leads to different provincial rankings of priority scores in different years.

[26] We obtained access to the scores in mock exams as well as the CEE in a high school of another province. Regression analysis suggest that the standard deviation of the mock exam-CEE score gap is roughly 32 points. Also, see here (in Chinese) for discussion regarding volatility of performance in the CEE. The online platform in the link is the Chinese counterpart of Quora.

To understand this, remember that the CEE consist of four subjects, and priority score is almost completely determined by summing up the *raw scores* of all subjects. However, the difficulty of subjects varies from year to year, such that the dispersion of students' performance does not move in the same direction, as demonstrated in Figure A6a. Given the unpredictability of subject difficulty, the way in which students' true academic ability is aggregated changes exogenously from year to year. For example, a student who is good at math may have a higher total score in a year where the math test is more difficult. To quantify the impact of such idiosyncratic aggregation, we regress rank-preserving score[27] on the polynomials of quantiles of each subject:

$$\text{Rank-Preserving Score} = f(Chinese, Math, English, Comprehensive)$$

If the regression is run within each year, the score should be perfectly predicted by the quantiles, which we confirm. If we run the regression over the whole sample (i.e. 2014-2018), quantiles cannot perfectly predict the score because of the exogenous change in score aggregation, as shown in Figure A6b. The residual is approximately normally distributed, creating additional variation in effective score that amounts to 5 points.

(II) Constructing share of college choice for small score bins In light of variation in scores that reflect orthogonal preference distribution, as well as the difficulty of analyzing the optimality of individual decisions in the presence of substantial heterogeneity, we take an approach different from Section 4 to analyze administrative data. Specifically, for any single college A, we compute the share of students choosing A as the first, second, third and fourth choices, respectively, within small intervals of priority scores. The idea is that the distribution of preferences can be regarded as roughly the same among students whose scores belong to adjacent bins, so that the share of choosing A can be viewed as data generated by a population of the same distribution of preferences. The length of score bins for A is 0.5 times the standard deviation of the cutoff of A, which amounts to $2 \sim 4$ points in priority score depending on the predictability of the cutoffs. We focus on the score bins that are within 1.5 standard deviations of the predicted mean of cutoffs[28] because only variation in scores that are close to the level of cutoffs would induce meaningful change in admission probability, which generates a sizable shock to choice patterns.

7.2 Testing Upward Movement in ROL: Data Analysis

If systematic upward movement exists, because the distribution of preferences is assumed to be the same in adjacent priority score bins, comparing the choice share data in pairs of adjacent bins would enable us to witness a chunk of DC types committing "upward movement": the share of students choosing A at a lower position in a bin will correlate with the share at a higher position in the right adjacent bin. Figure 7d presents an example, where upward movement leads to positive

[27] Converting raw CEE score to its 2018 rank-preserving equivalent, as in Section 2.

[28] Six bins are included: [Mean-1.5SD,Mean-1SD], [Mean-1SD,Mean-0.5SD], [Mean-0.5SD,Mean], [Mean,Mean+0.5SD], [Mean+0.5SD,Mean+1SD], [Mean+SD,Mean+1.5SD].

correlation between bottom left bins and their top right adjacent counterparts, as connected by the arrows that indicate the direction of movement. In contrast, when downward movement happens, the correlations between bottom left bins and their top right adjacent counterparts are negative, as shown in Figure 7c.

This observation motivates us to conduct the following empirical exercise. Suppose the share of students choosing A to be their xth choice in score bin s is $\delta^x_{s,A}$. For any $x' = x + 1$, we run the following regression:

$$\delta^x_{s,A} = \gamma \delta^{x'}_{s-1,A} + FE_A + FE_s \quad (3)$$

where FE_A is college fixed effects that control for horizontal preferences, and FE_s is score bin fixed effects that control for admission probability.

Substantial upward movement implies that the choice share in score bins $\delta^x_{s,A}$ positively correlates with the choice share in its top right adjacent counterparts $\delta^{x'}_{s-1,A}$. Thus three tests can be conducted: (i) $x = 1, x' = 2$; (ii) $x = 2, x' = 3$; (iii) $x = 3, x' = 4$. $\gamma > 0$ implies that a substantial share of students commits upward movements, a pattern that is only consistent with the DC decision rule.

This test does not place any parametric assumptions on preferences. Neither does it rely on a specific level of admission probability to identify the presence of the DC type because score-bin fixed effects have been included, which excludes patterns that are driven by risk preference or over/under-confidence. What it tests is the comparative statics concerning movements along the list that are induced by *variation* in admission probability.

The nonparametric nature of this test does come at a cost. As upward movement and downward movement usually coexist, γ cannot be interpreted as a function of share of the DC Type, and, in this one-sided test, negative γ does not imply the non-existence of the DC type.

We report the results in Table 4. Panel A shows that the estimated $\hat{\gamma}$ are significantly positive at all positions, albeit with different levels ($x = 1, x' = 2$: 0.151, SE=0.027; $x = 2, x' = 3$: 0.679, SE=0.057; $x = 3, x' = 4$: 0.249, SE=0.072). Panels B, C, and D show that the significance is robust to how the score bins are constructed, though the exact magnitude is slightly unstable for the case of $x = 2, x' = 3$. These results suggest that the presence of the DC type is so substantial that upward movements emerge as a dominant pattern in this choice share data.

8 Structural Estimation using Administrative Data

Sections 4, 5, 6, and 7 have established the presence of DC types in our sample. However, reduced-form analysis, while straightforward, does not tell us exactly how many students in the administrative data are actually using the DC decision rule. This effort is complicated by the complexity of the portfolio choice problem over a large number of choices whose preferences are unobservable and heterogeneous. As Agarwal and Somaini (2019) points out, to identify sub-optimality in ROLs, administrative data alone is usually insufficient. However, the unique upward movement in our setting provides us a glimpse of hope. It shows that non-parametrical administrative data alone is sufficient in our context to tell the DC type apart from its rational counterparts. In light of this message, in this section we implement a parametrized structural model to estimate the share of the DC type. We then discuss the welfare implication of de-biasing and alternative mechanisms.

8.1 Setup

Structure of College Preferences We allow for substantial, flexible preference heterogeneity in our estimation. For individual i, the utility of admission to college j is

$$u_{ij} = f(\theta_i^C, C_{ij}, SES_i) + g(\theta_i^d, d_j, SES_i) + h(\theta_i^X, X_j, SES_i) + O_i + \epsilon_{ij}$$

where C_{ij} is the competitiveness of college j with respect to student i, SES_i is the average educational attainment in student i's township of residence. Function $f(.)$ controls the curvature over college preferences and takes the form of the CRRA function, with the curvature parameter being θ_i^C[29]. θ_i^C is normally distributed with unknown variance, and the mean is allowed to vary across students of different SES levels. We normalize d_j as the distance between students' home and the location of the college. Note that subscript i is omitted. Since all but two first-tier colleges are located outside Ningxia, and are clustered in metropolitan areas far away, the distances barely differ for students living in different areas in Ningxia. Function $g(.)$ controls preferences for distance and is quadratic with parameter vector θ_i^d. θ_i^d is jointly normally distributed with unknown diagonal variance matrix, and the mean is allowed to vary across students of different SES levels. $h(\theta_i^X, X_j, SES_i)$ controls the interaction between other college characteristics and students' socioeconomic status, with the interaction parameter θ_i^X permitted to be normally distributed, with unknown variance and SES-specific mean. O_i measures the desirability of first-tier colleges overall relative to outside options. $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$ is the individual-college specific shock to admission utility.

Mixture Model We estimate a mixture model where there are two types of students: DC and rational. In this mixture model, we assume that share of the DC type among students is a function of the SES status of student i:

$$P(\text{DC Type}|\text{SES}_i) = \frac{exp(\gamma_0 + \gamma_1 * SES_i)}{1 + exp(\gamma_0 + \gamma_1 * SES_i)}$$

Admission Probability In the benchmark estimation, we use the estimated probability $\hat{p_{ij}}$ as in Section 3.2.

[29] As in Section 4.2, we ensure that C_{ij} is positive by taking the difference between itself and the minimally competitive college.

8.2 Estimation Strategy and Identification

Following Section 4, we focus on students whose priority score percentile is above 40% because the identification of departure from the optimal benchmark comes from the coexistence of reversals and cautiousness. As discussed in Section 4.1, these are more pronounced among students whose priority score is high enough such that college choices become less binding. We split our sample according to students' CEE score and conduct the estimation for 40% $\sim$ 60%, 60% $\sim$ 80%, 80% $\sim$ 100% separately because those of different levels of academic ability tend to choose colleges of different levels of competitiveness, as demonstrated in Figure A7. Thus, to minimize the impact of unobserved preference heterogeneity it is better to estimate among students of similar level of priority scores.

We use Simulated Method of Moments to estimate this model because the large set of potential portfolios renders simulated maximum likelihood computationally impossible. In this parametrized model, the moments on characteristics of the listed colleges (physical distance, as well as share of choosing a specific type of college) jointly identify the distribution of choice over colleges, hence the horizontal preferences over observables. Curvature over competitiveness is identified by the mean of admission probability of chosen colleges and competitiveness. Other horizontal preferences are assumed to be idiosyncratic, and thus bounded by competitiveness (i.e., cutoffs) of the choice.

Importantly, based on the discussion about upward movements in Section 5.4, all the moments are constructed for the first, second, third and fourth choices separately. We argue that it is these position-specific moments that identify the DC type. Moreover, informed by our findings in Section 4.3, the moments are constructed separately for four SES quartiles to examine whether the share of the DC type is higher among the socioeconomically disadvantaged compared to their counterparts.

We simulate each students' choices 50 times, and calculate the simulated moments by averaging across different rounds of simulation for all students in each SES*Score cell. The estimation minimizes the weighted distance between simulated moments $m(\theta)$ and data m_0:

$$min\ (m(\theta) - m_0)' W (m(\theta) - m_0)$$

The estimator achieves asymptotic normality with the estimated variance being:

$$(\hat{G}' W \hat{G})^{-1} (\hat{G}' W (1 + \frac{1}{50})(\hat{\Omega}/N) W \hat{G})(\hat{G}' W \hat{G})^{-1}$$

where 50 corresponds to the number of simulated choices for each observation (Laibson et al., 2007; DellaVigna et al., 2016), $\hat{G} \equiv \frac{\partial m(\theta)'}{\partial \theta}$ and $\hat{\Omega} = Var(m(\hat{\theta}))$. In our estimation, to enhance the efficiency of estimation, W is selected to be the inverse of the covariance matrix.

8.3 Estimation Results

Table 5 compares the performance of a model that only allows for the rational type to a mixture model that allows for both the rational type and the DC type. While all models are over-identified and rejected, the mixture model decreases the distance by 42.0%, 69.1%, 53.3% for the 40% $\sim$ 60%, 60% $\sim$ 80%, 80% $\sim$ 100% subsamples, respectively. This improvement is substantial and MMSC-BIC metric[30] (Andrews and Lu, 2001), an analogue of Bayesian Information Criterion, favors the mixture model over rational benchmark as well.

The estimated share of the DC type is 53.1% (SE=0.61%), 45.1% (SE=0.54%), 55.1% (SE=0.55%) for 40% $\sim$ 60%, 60% $\sim$ 80%, 80% $\sim$ 100% subsamples, respectively. These estimates are interestingly close to the estimate we get from the online survey (48.7%, SE=1.8%). The estimates on the marginal effect of SES are negative, where 1 standard deviation of decrease in SES index is associated with a decrease of 3.71% (SE=0.57%), 3.68% (SE=0.56%), 6.02% (SE=0.55%) in propensity of being a DC type. This negative effect is less than the estimated effect from the survey (7.3%, SE=1.7%), though the estimates are not statistically different.

The estimated curvature over competitiveness is mild and sometimes positive, ranging from -0.486 to 0.413 (ρ in CRRA specification), with all standard errors below 0.1. A general pattern is that, when only the rational type is allowed, the estimated curvature is larger than when both types are present. This suggests that the model does try to rationalize the cautiousness by increasing curvature. However, even in this case the curvature is mild ($\rho < 0.5$), which suggests that increasing curvature does not further improve fit despite its ability to generate cautiousness, because an extremely curved utility function will generate implausible choice patterns over other observables. For the mixture model, the curvature is in the negative territory, suggesting that top colleges are highly sought after. In summary, despite the failure to rationalize cautiousness in the one-type rational model, the levels of estimated curvature in both models are consistent with anecdotal evidence on the perceived importance of competitiveness/cutoffs on college prestige.

Out-of-sample prediction of the mixture model is also substantially better than the one-type rational model. The structural estimation only uses the data from 2015, 2017, and 2018, because we have access to township-level student addresses only in these years to measure SES in a more precise way. The measure of SES in 2014 and 2016 is county-level adult educational attainment, which is substantially less precise, and thus is left out of the estimation and used for out-of-sample testing. As demonstrated in Panel B of Table 5, compared to the one-type rational model, the mixture model decreases the distance 14.9%, 43.4%, and 47.2%, respectively, and fits much better in the moments that are related to our predictions about cautiousness and reversals.

Quantitatively, the contribution of the DC decision rule to the fit is more robust across models that allow for different degrees of preference heterogeneity. To quantify this point, Panel A reports specifications where we keep all model setup the same as in Table 5, except that we assume away across-type preference heterogeneity and take into account only preferences over competitiveness and distance. The numbers of parameters for each subsample, as a result, drop from 20, 29 to 8, 11, respectively. We find that the fit with the DC type, despite having merely three more parameters, still improves the fit by 60.6%, 54.6%, and 57.2%, respectively. More strikingly, if

[30]The formula for this metric is $d - (m - p)ln(n)$, where d is the distance, m is the number of moments, p is the number of parameters, and n is sample size. The criteria favor smaller value.

we compare the even columns in Panel A, Table 6 to the odd columns in Table 5, the fit of the mixture model, despite having less flexibility in preferences and fewer parameters, is also much better than the one-type model in benchmark estimation (distance decreases by 40.4%, 40.5%, and 34.5%, respectively). The estimates on the share of the DC type remain precisely estimated and stable, where the estimated share is 49.3% (SE=0.25%), 47.8% (SE=0.23%), 54.0% (SE=0.30%) respectively.

To further understand how the model fits the data, we look at the key moments that identify the presence of the DC type. Figure 8 compares the overall data and fit from the one-type rational model and the mixture model, for average admission probability (Figure 8a) and share of reversals with different thresholds (Figure 8b), respectively. While both the one-type rational model and mixture model generate substantial reversals by introducing heterogeneous preferences, the one-type model fails to explain the cautiousness in the first choices by a fairly large margin (20%), while it predicts extreme cautiousness (probability $> 99\%$) for the fourth choices, contrary to the data, in which the mean of probability is around 90% for the fourth choices. Moreover, the one-type model also fails to match the share of reversals when the threshold is low, which further differentiates itself from the mixture model in terms of fit. Under this parametrization, an implausible level of curvature or horizontal preferences would substantially worsen the fit of other moments. As an example of how arbitrary preferences could worsen the fit overall, Figure 8c shows that both models yield a correlation of 0.9, whereas in Section 4.2 we show that simulated correlation decreases to 0.8 or even lower if we introduce more curvature or horizontal preferences. While our model does not directly fit the empirical test on upward movements, Figure 7d shows that, consistent with our theory, the upward movement coefficient γ is negative for data simulated by the rational one-type model, and the coefficients of the mixture model are higher and in some cases positive. Consequently, the mixture model generates upward movements that are closer to the patterns we observe in the data.

In Figure 9 we compare the prediction of the mixture model for the most and least advantaged quartile to the data. Similarly to the empirical exercise in Section 4, we reweight to control for priority scores. While the fit is not perfect, the comparative statics are consistent with our expectation. The predicted cautiousness for the first choices is indeed more evident among the disadvantaged, and the predicted gap is less for other choices, consistent with the data. In terms of reversal, the predicted share is higher among the disadvantaged, in line with the data.

8.4 Welfare

We compare the admission outcomes in our data to a counterfactual where all the behavioral applicants have been de-biased and play optimally, as well as counterfactual scenarios where behavioral applicants continue to use the DC decision rule under other mechanisms. This investigation is helpful for the analysis of the distributional consequences of suboptimal decision-making (Pathak and Sönmez, 2008), and potentially helpful for policy discussion. To compute the counterfactual, we follow Kapor et al. (2020) to simulate how everyone, if rational, would react to the current equilibrium (distribution of cutoffs). We thereby obtain new lists. Then, the program uses the new lists to simulate the cutoffs, and iterate until the cutoffs converge.

The money metrics of welfare is motivated by the analogy that students' priority scores (determined by their exam performance, not college demand or their strategy) serve as their WTP for college education, and that cutoffs of the colleges serve as the prices of such service. We treat students' priority scores as students "budget set", and measure the welfare change as the equivalent variation (EV) in terms of priority score. In other words, for each individual applicant, EV is the amount of change in its priority score in the current equilibrium that results in the same change in expected utility had the score remain unchanged but a new equilibrium happened. We take this approach for two reasons. First, the estimation results suggest that priority score is an effective measure of student ability, and cutoff is a good predictor of college quality. Second, common measures such as physical distance contradict strong geographical preferences toward economically developed area in China, which are all very far from Ningxia.

De-Biasing If all students had the same cardinal preferences, the matching process would be a zero-sum game. Conventional wisdom suggests that the sophisticated usually take advantage of the naive in a market setting (Gabaix and Laibson, 2006; Pathak and Sönmez, 2008). However, even without horizontal preferences, the intensity of vertical preferences may actually differ, and acting strategically in the correct way will help students communicate such intensity and improve equilibrium outcomes (Abdulkadiroğlu et al., 2011). It is an empirical question whether students' preference profile in our case would lead to different distributional implications of de-biasing. We report our finding in Panel C of Table 5. De-biasing improves the 3rd, 4th and 5th quintile of the DC type's welfare, which is equivalent to an improvement of 0.495, 0.253, 0.082 of a standard deviation (s.d.) of the priority score in the old equilibrium. Interestingly, de-biasing also increases the welfare of the rational type whose priority scores belong to the 3rd and 4th quintile by 0.368 and 0.217 s.d. of the score, and decreases the welfare of those whose scores belong to the highest quintile by 0.080. These result suggest that, far from a zero-sum game, de-biasing benefits the vast majority of students except for the most advantaged rationals at the top of priority score distribution. Intuitively, this happens because, in the old equilibrium, the behavioral type creates a mismatch effect by affecting the rational counterparts whose priority score is lower. De-biasing eliminates this effect, and benefits any rational type whose score is not high enough that he or she is immune to the impact of the mistaken choices due to behavioral type decision-making.

DAA with Unlimited List DAA that allows applicants to list an unlimited number of choices restores the strategy-proof property of the mechanism, such that the mistakes in strategic play are no longer relevant. However, it has an additional effect that students can no longer express their preference intensity through strategic play. We report the welfare effect of adopting a strategy-proof property in Panel C. A DAA mechanism with an unlimited list decreases the welfare of the rationals in the 3rd, 4th, and 5th quintiles by 0.329, 0.725, and 0.045 s.d. of score. It increases the welfare of the DC type in the 3rd and 4th quintile by 0.231 and 0.447 s.d. of score, but decreases the welfare of the DC in the 5th quintile by 0.155 s.d. of score. Overall, switching to a strategy-proof mechanism is not welfare enhancing for the majority of the students. Note that empirically the finding here resembles previous studies (Agarwal and Somaini, 2018; Calsamiglia et al., 2020). An important caveat is that such evaluation ignores a potential benefit that, under unconstrained DAA, mistaken beliefs about admission probability are no longer costly (Kapor et al., 2020).

Manipulable Mechanisms Compared to the constrained DAA in our setting, the Boston mechanism is more manipulable. Different from the case of Pathak and Sönmez (2008), however, the behavioral bias we document is less costly under the Boston mechanism because it weakens the rational type's advantage of using backup choices as "insurance" to act aggressively for their top choices. Because the behavioral type doesn't know how to take this advantage, switching to Boston mechanism might decrease the welfare inequality between these two types. However, as reported in Panel C, our results suggest that switching to the Boston mechanism leads to a decrease in welfare ($0.02 \sim 0.15$ s.d. of score) for students of both types, in the 3rd, 4th and 5th quintiles, although the decrease for the DC type is indeed smaller than the decrease for the rational type. Together with the results from DAA with an unlimited list, our analysis provides empirical support for Chen and Kesten (2017)'s theoretical finding that the Chinese parallel mechanism, as a middle ground between the Boston and DAA mechanism, may be better than both in our context, despite a substantial share of applicants who do not understand the option value brought about by the backup bottom choices. This is because the mechanism in our setting reduces the cost of miscoordination (Chen and Kesten, 2019), while preserving students' ability to communicate their preference intensities via strategic play.

Inequality Figure 10 presents how de-biasing and mechanism switching differentially affect students of different socioeconomic status. Switching to an unconstrained Deferred Acceptance Algorithm or a Boston mechanism decreases the average welfare of students of every socioeconomic status. In contrast, de-biasing increases the welfare of all socioeconomic levels, where the welfare increase for the most disadvantaged quartile is equivalent to 0.294 s.d. of score, and the welfare increase for the most advantaged quartile is equivalent to 0.221 s.d. of score. This finding could be explained by the fact that the estimated share of the DC type is higher among the disadvantaged. Consequently, given the estimated preferences, de-biasing outperforms alternative mechanisms in both equity and efficiency.

9 Alternative Considerations

9.1 Subjective Beliefs

Evidence from Survey Kapor et al. (2020) documents that students may not accurately estimate the probability of admission, especially when the structure of priority scores is more complicated. While in our context the only uncertainty comes from the variation in cutoffs, a one-dimensional object, it is quite unlikely that students' beliefs are perfectly accurate. We elicited students' beliefs regarding the unconditional probability of the four colleges on their lists, and compare the elicited beliefs to the estimated probability that we construct. Subjective beliefs are positively correlated with estimated probability, at a correlation of 0.5034. We define belief error as the difference between subjective beliefs and estimated probability, and regress the error on students' socioeconomic index, controlling for other variables including priority score and college preferences. As demonstrated in Table B5, students on average overestimate the chance of admission by 11.3%, and the mean of absolute error is 30.6%.

Accounting for Heterogeneous Beliefs in Structural Estimation The specification of the perceived probability admission of college j for student i is:

$$p_{ij} = max\{0, min\{1, \hat{p_{ij}} + \tau_{ij}\}\}$$

where $\tau_{ij} \sim N(\mu_{\tau 0} + \mu_{\tau 1}\text{SES}_i, exp(\mu_{\tau 2} + \mu_{\tau 3}\text{SES}_i)^2)$. $\mu_{\tau 0} + \mu_{\tau 1}\text{SES}_i$ dictates student i's overall optimism; $exp(\mu_{\tau 2} + \mu_{\tau 3}\text{SES}_i)^2$ dictates the standard deviation of student i's idiosyncratic beliefs about admission probability on top of the student's overall optimism. In the estimation, we calibrate the parameter by referring to the estimation results from survey. Specifically, the value of $\mu_{\tau 0} =$ 11.3% and $\mu_{\tau 1} = 2.17\%$ is taken from results in Table B5. $\mu_{\tau 2} = -0.896$ and $\mu_{\tau 3} = 0$ is based on the variance of beliefs and the finding that belief error in its square term does not change significantly across students of different SES status.

In Panel B of Table 6, we consider the impact of heterogeneous beliefs about admission probability on our estimation. We re-run the same specifications in Table 5. In terms of fit, the mixture model outperforms the one-type rational model by an even larger margin (54.3%, 85.8%, 93.9% for the 40% $\sim$ 60%, 60% $\sim$ 80%, 80% $\sim$ 100% respectively), with the MMSC-BIC metric favoring the mixture model even more. The fit with heterogeneous beliefs is not as good as our benchmark estimate in Table 5, potentially because our model of belief errors is unable to perfectly capture students' beliefs. Another surprising finding is that the estimated share of the DC type is more than 90% regardless of the subsample we focus on. The overall overconfidence and substantial idiosyncrasies in beliefs ensure that students have large positive belief errors about many colleges, such that, even if students eliminate the risk by only choosing colleges whose subjective probability is close to 100%, the objective probability may actually be much lower. This is particularly a problem for the rational models, as the first choices under the rational rule are the most preferred, and thus more likely to be the competitive colleges mistakenly chosen due to large positive belief errors.

9.2 Correlation in Admission Events

If the probability of meeting the cutoff of one college is correlated with meeting that of another, our assumptions about independence are defied. In this case, assuming independence of admission probability alone may result in suboptimal portfolio choices (Shorrer, 2019; Rees-Jones et al., 2020).

Unlike the setting in the aforementioned studies, however, in our case students already know their priority score and ranking by the time of application.

To test the presence of pairwise correlation, we conduct an empirical test with the same specification as we used in Section 3.2 on the administrative dataset. The difference is that, in this new empirical exercise, the dependent variable is students' second choices, and the regression is run on those who were not admitted by their first choice:

$$1(\text{Admitted to Second Choices})_i = \alpha_2 + \beta_2 \hat{p}_{ij}$$

If the admission to the first choices does not correlate with the admission probability of the second choices, we would expect $\alpha_2 = 0$ and $\beta_2 = 1$, which is the null hypothesis of this exercise.

The estimation results suggest that our estimates of admission probability remain accurate conditional on the rejection of the first choices, suggesting that in our setting pairwise correlation does not significantly alter admission probability. As shown in Columns 3 and 4 of Table B2, $\hat{\alpha_2} = 0.0033$ (SE=0.0037) and -0.0094 (SE=0.0060) for the science and humanity tracks, respectively, whereas $\hat{\beta_2} = 0.9883$ (SE=0.0060) and 1.0252 (SE=0.0107) for the science and humanity tracks, respectively. The p-values of the F-test are 0.085 and 0.059, respectively, meaning that we fail to reject the null hypothesis.

9.3 College Preferences vs. Major Preferences

Major preference does not affect assignment of college; essentially the Chinese system is a "college-then-major" system(Chen and Kesten, 2017; Calsamiglia et al., 2020). Major studies typically begin in the second year of bachelor education, and since the 2010s, the Ministry of Education (MoE) of PRC has successfully pushed for a lower barrier in major switching [31]. To assess whether major concerns affect risk taking, we asked students which factor they were most concerned about in college applications. We report the relevant statistical analysis in Table B6. In Columns 1 and 2, we regress the dummy indicating whether students consider major to be their top concern on a normalized SES index. Only 13.7% of the survey takers consider major to be their top concern, and the share is slightly lower among the advantaged students.

Consideration of major could affect students' strategy if students think ahead, and want to outcompete their peers who are admitted to the same college in terms of academic ability. To examine its quantitative impact on risk-taking, we regress the estimated unconditional probability of the first choices (Columns 3-4) and beliefs about being admitted to the first choices (Columns 5-6) on those who consider major to be of top concern. As expected, students tend to be more cautious if they consider major to be of top concern, but its quantitative impact on the probability of first choices is less than 10% compared to those who do not regard major as their top consideration. We calibrate its impact on the full sample average of first choice probabilities by calculating the product of the average impact due to major concerns and the share of students who consider major

[31] The link here is an example. Since this campaign by the MoE, major distinctions have become more coarse and less of a concern to students.

to be important. Reassuringly, it contributes at most a 1.2% increase in the mean unconditional probability of the first choices.

10 Discussion and Conclusion

We provided evidence that, when a centralized admission system incentivizes strategic reporting, such that the optimal decision rule is complex, a substantial share of student applicants engage in a myopic strategy of risk taking in a way that can be well explained by the theory of directed cognition. Our estimation also suggests that the DC type is particularly concentrated among disadvantaged students, and is responsible for the worse admission outcomes among the disadvantaged compared to their advantaged counterparts who achieve the same level of CEE score, exacerbating inequality in educational attainment. In a counterfactual analysis, de-biasing the DC type substantially increases the welfare for the behavioral type and, surprisingly, for the majority of the rational type, by enhancing matching efficiencies; it outperforms alternative mechanisms (e.g. Boston and Deferred Acceptance mechanism without list constraint) in terms of both equity and efficiency.

The DC decision rule could be applied to other contexts or mechanisms where the optimal response necessitates processing risk and uncertainty that unravel in multiple stages. Our paper provides empirical methods to identify systematic departures from optimality for ROL data, where heterogeneous preferences typically make such investigation difficult. Our findings suggest that behavioral bias plays a role in inequality formation. To address such imperfections in decision making, it may sometimes be more desirable to intervene against the bias itself, rather than changing the mechanism altogether. We hope that more empirical research will be conducted in other contexts to uncover the broad picture of potential sub-optimality in a complex decision-making environment.

References

Abdulkadiroğlu, Atila, Yeon-Koo Che, and Yosuke Yasuda (2011) "Resolving conflicting preferences in school choice: The" boston mechanism" reconsidered," *American Economic Review*, Vol. 101, pp. 399–410.

Abeler, Johannes and Simon Jäger (2015) "Complex tax incentives," *American Economic Journal: Economic Policy*, Vol. 7, pp. 1–28.

Agarwal, Nikhil and Paulo Somaini (2018) "Demand Analysis using Strategic Reports: An application to a school choice mechanism," *Econometrica*, Vol. 86, pp. 391–444.

——— (2019) "Revealed Preference Analysis of School Choice Models," *Annual Review of Economics*, Vol. 12.

Ajayi, Kehinde F (2013) "School choice and educational mobility: Lessons from secondary school applications in Ghana," *Institute for Economic Development Working Paper*, Vol. 259.

Allcott, Hunt, Benjamin B Lockwood, and Dmitry Taubinsky (2019) "Regressive sin taxes, with an application to the optimal soda tax," *The Quarterly Journal of Economics*, Vol. 134, pp. 1557–1626.

Andrews, Donald WK and Biao Lu (2001) "Consistent model and moment selection procedures for GMM estimation with application to dynamic panel data models," *Journal of econometrics*, Vol. 101, pp. 123–164.

Artemov, Georgy, Yeon-Koo Che, and Yinghua He (2017) "Strategic 'mistakes': Implications for market design research," *NBER Working Paper*.

Bertrand, Marianne, Sendhil Mullainathan, and Eldar Shafir (2006) "Behavioral economics and marketing in aid of decision making among the poor," *Journal of Public Policy & Marketing*, Vol. 25, pp. 8–23.

Bhargava, Saurabh, George Loewenstein, and Justin Sydnor (2017) "Choose to lose: Health plan choices from a menu with dominated option," *The Quarterly Journal of Economics*, Vol. 132, pp. 1319–1372.

Bhargava, Saurabh and Dayanand Manoli (2015) "Psychological frictions and the incomplete take-up of social benefits: Evidence from an IRS field experiment," *American Economic Review*, Vol. 105, pp. 3489–3529.

Brot-Goldberg, Zarek C, Amitabh Chandra, Benjamin R Handel, and Jonathan T Kolstad (2017) "What does a deductible do? The impact of cost-sharing on health care prices, quantities, and spending dynamics," *The Quarterly Journal of Economics*, Vol. 132, pp. 1261–1318.

Calsamiglia, Caterina, Chao Fu, and Maia Güell (2020) "Structural estimation of a model of school choices: The Boston mechanism versus its alternatives," *Journal of Political Economy*, Vol. 128, pp. 642–680.

Camerer, Colin F, Eric Johnson, Talia Rymon, and Sankar Sen (1993) "Cognition and framing in sequential bargaining for gains and losses," *Frontiers of game theory*, Vol. 104, pp. 27–47.

Campbell, John Y (2016) “Restoring rational choice: The challenge of consumer financial regulation,” *American Economic Review*, Vol. 106, pp. 1–30.

Chade, Hector and Lones Smith (2006) “Simultaneous search,” *Econometrica*, Vol. 74, pp. 1293–1307.

Chen, Yan and Onur Kesten (2017) “Chinese college admissions and school choice reforms: A theoretical analysis,” *Journal of Political Economy*, Vol. 125, pp. 99–139.

——— (2019) “Chinese college admissions and school choice reforms: An experimental study,” *Games and Economic Behavior*, Vol. 115, pp. 83–100.

Chen, Yan and Tayfun Sönmez (2002) “Improving efficiency of on-campus housing: An experimental study,” *American economic review*, Vol. 92, pp. 1669–1686.

Chetty, Raj, Adam Looney, and Kory Kroft (2009) “Salience and taxation: Theory and evidence,” *American economic review*, Vol. 99, pp. 1145–77.

Congdon, William J, Jeffrey R Kling, and Sendhil Mullainathan (2009) “Behavioral economics and tax policy,” *National Tax Journal*, Vol. 62, pp. 375–386.

De Haan, Monique, Pieter A Gautier, Hessel Oosterbeek, and Bas Van der Klaauw (2015) “The performance of school assignment mechanisms in practice.”

DellaVigna, Stefano (2018) “Structural behavioral economics,” in *Handbook of Behavioral Economics: Applications and Foundations 1*, Vol. 1: Elsevier, pp. 613–723.

DellaVigna, Stefano, John A List, Ulrike Malmendier, and Gautam Rao (2016) “Voting to tell others,” *The Review of Economic Studies*, Vol. 84, pp. 143–181.

Dreyfuss, Bnaya, Ori Heffetz, and Matthew Rabin (2019) “Expectations-based loss aversion may help explain seemingly dominated choices in strategy-proof mechanisms,”Technical report, National Bureau of Economic Research.

Dynarski, Susan M and Judith E Scott-Clayton (2006) “The cost of complexity in federal student aid: Lessons from optimal tax theory and behavioral economics,” *National Tax Journal*, Vol. 59, pp. 319–356.

Gabaix, Xavier and David Laibson (2006) “Shrouded attributes, consumer myopia, and information suppression in competitive markets,” *The Quarterly Journal of Economics*, Vol. 121, pp. 505–540.

Gabaix, Xavier, David Laibson, Guillermo Moloche, and Stephen Weinberg (2006) “Costly information acquisition: Experimental analysis of a boundedly rational model,” *American Economic Review*, Vol. 96, pp. 1043–1068.

Hassidim, Avinatan, Assaf Romm, and Ran I Shorrer (2016) “” Strategic” Behavior in a Strategy-Proof Environment,” in *Proceedings of the 2016 ACM Conference on Economics and Computation*, pp. 763–764.

Jia, Ruixue and Hongbin Li (2016) “Access to elite education, wage premium, and social mobility: the truth and illusion of China’s college entrance exam,” *Stanford Center for International Development, Working Paper*.

Johnson, Eric J, Colin Camerer, Sankar Sen, and Talia Rymon (2002) “Detecting failures of backward induction: Monitoring information search in sequential bargaining,” *Journal of Economic Theory*, Vol. 104, pp. 16–47.

Kapor, Adam J, Christopher A Neilson, and Seth D Zimmerman (2020) “Heterogeneous beliefs and school choice mechanisms,” *American Economic Review*, Vol. 110, pp. 1274–1315.

Laibson, David, Andrea Repetto, and Jeremy Tobacman (2007) “Estimating discount functions with consumption choices over the lifecycle,”Technical report, National Bureau of Economic Research.

Li, Shengwu (2017) “Obviously strategy-proof mechanisms,” *American Economic Review*, Vol. 107, pp. 3257–87.

Lucas, Adrienne M and Isaac M Mbiti (2012) “The determinants and consequences of school choice errors in Kenya,” *American Economic Review*, Vol. 102, pp. 283–88.

Martínez-Marquina, Alejandro, Muriel Niederle, and Emanuel Vespa (2019) “Failures in Contingent Reasoning: The Role of Uncertainty,” *American Economic Review*, Vol. 109, pp. 3437–74.

Pathak, Parag A and Tayfun Sönmez (2008) “Leveling the playing field: Sincere and sophisticated players in the Boston mechanism,” *American Economic Review*, Vol. 98, pp. 1636–52.

Rabin, Matthew and Georg Weizsäcker (2009) “Narrow bracketing and dominated choices,” *American Economic Review*, Vol. 99, pp. 1508–43.

Rees-Jones, Alex, Ran Shorrer, and Chloe J Tergiman (2020) “Correlation Neglect in Student-to-School Matching,”Technical report, National Bureau of Economic Research.

Rees-Jones, Alex and Samuel Skowronek (2018) “An experimental investigation of preference misrepresentation in the residency match,” *Proceedings of the National Academy of Sciences*, Vol. 115, pp. 11471–11476.

Rees-Jones, Alex and Dmitry Taubinsky (2020) “Measuring “schmeduling”,” *The Review of Economic Studies*, Vol. 87, pp. 2399–2438.

Roth, Alvin E and Elliott Peranson (1999) “The redesign of the matching market for American physicians: Some engineering aspects of economic design,” *American economic review*, Vol. 89, pp. 748–780.

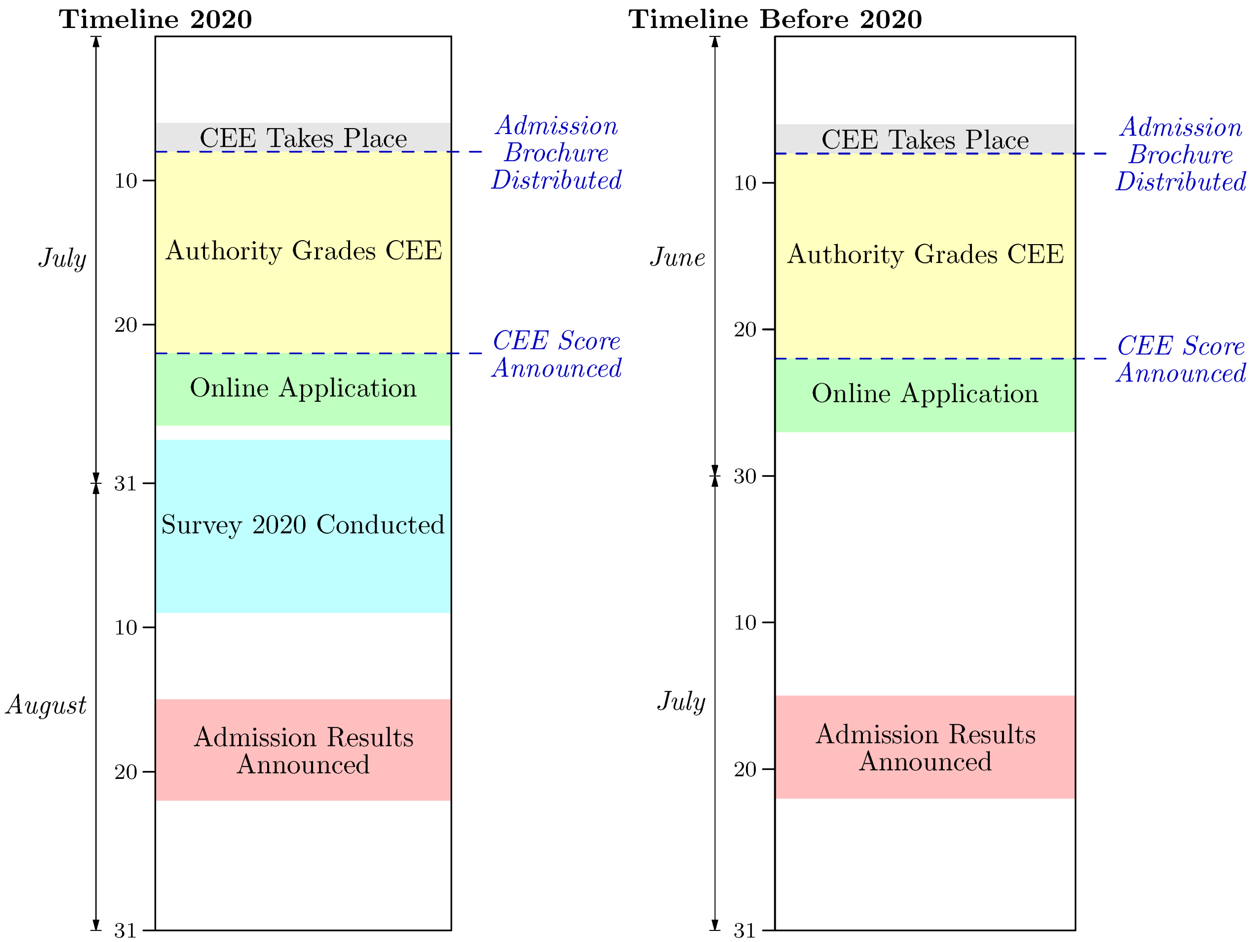
Shorrer, Ran I (2019) “Simultaneous search: Beyond independent successes,” in *Proceedings of the 2019 ACM Conference on Economics and Computation*, pp. 347–348.

Shorrer, Ran I and Sándor Sóvágó (2018) “Obvious mistakes in a strategically simple college admissions environment: Causes and consequences,” *Available at SSRN 2993538*.

Von Gaudecker, Hans-Martin, Arthur Van Soest, and Erik Wengstrom (2011) “Heterogeneity in risky choice behavior in a broad population,” *American Economic Review*, Vol. 101, pp. 664–94.

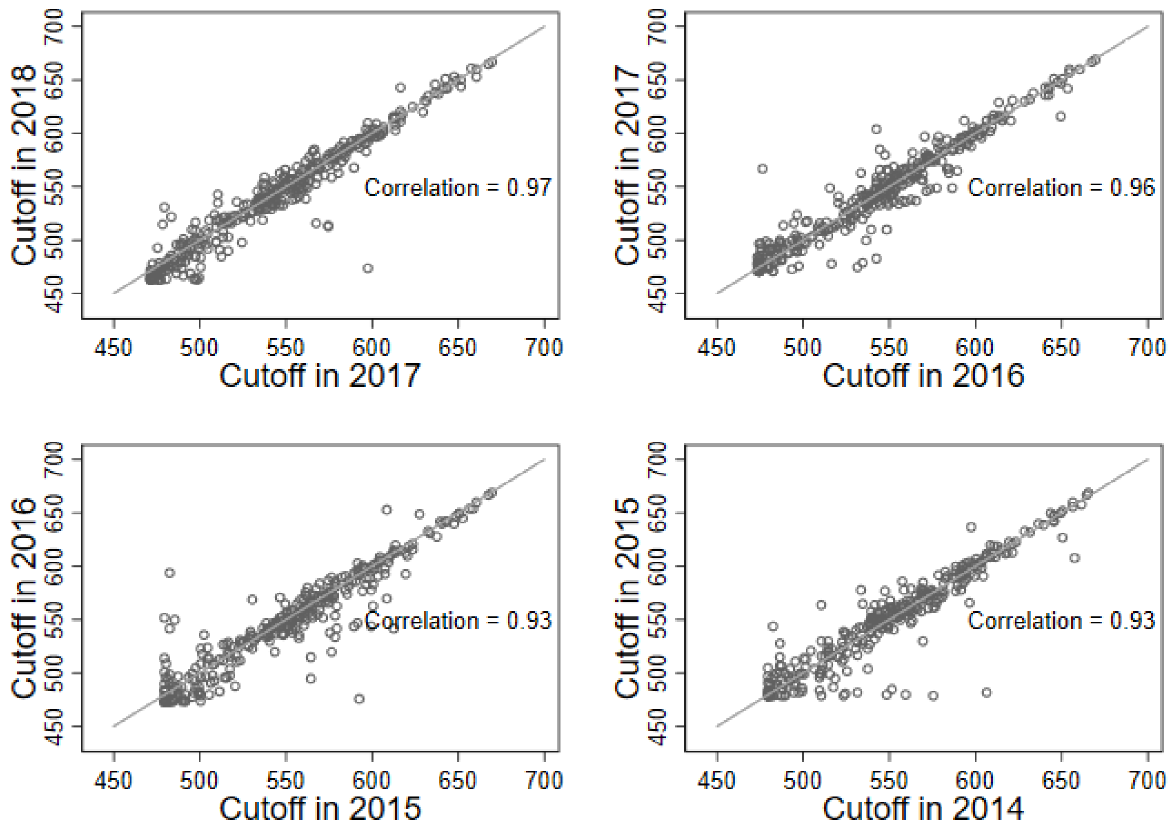
Walters, Christopher R (2018) “The demand for effective charter schools,” *Journal of Political Economy*, Vol. 126, pp. 2179–2223.

Figure 1: Timeline of College Admission Process



Note: This figure presents the timeline of college admission process, as detailed in Section 2.1, 2.2, 2.3. Note that the timeline remains unchanged until 2020, when the college entrance exam (CEE) was postponed by exactly one month due to COVID-19. As a result, all the admission procedures thereafter were postponed by exactly one month as well. As shown in the 2020 timeline, the survey was conducted right after application deadline, but before students were informed of their admission outcomes.

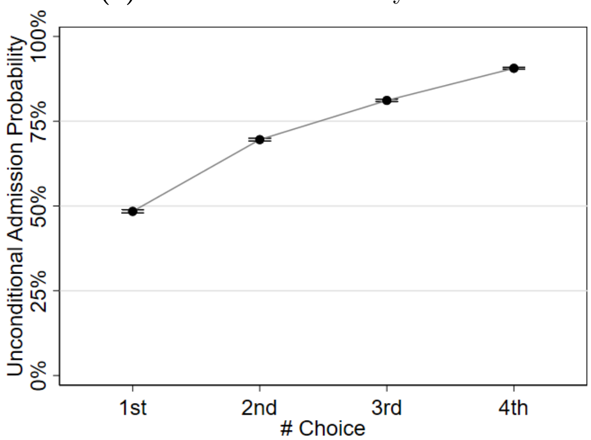
Figure 2: Admission Cutoffs Across Years



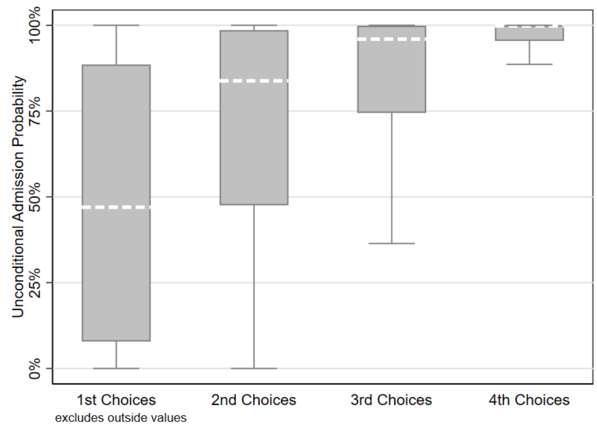
Note: This figure plots the admission cutoffs for all colleges that admit Ningxia students during 2014-2018, for the discussion in Section 3.2. The admission cutoffs are converted to its 2018 rank-preserving equivalents. Each dot represents the admission cutoffs of a specific college in two consecutive years. Top left graph plots the cutoffs in 2018 against 2017. Similarly, top right graph plots the cutoffs in 2017 against 2016; bottom left graph plots the cutoffs in 2016 against 2015; bottom right graph plots the cutoffs in 2015 against 2014.

Figure 3: Summary Statistics of Unconditional Admission Probability

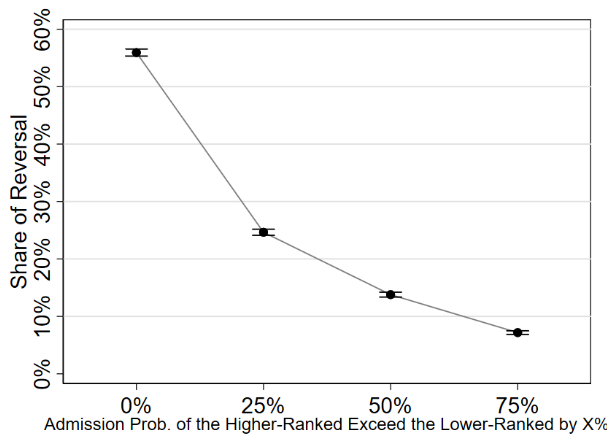
(a) Admission Probability: Mean



(b) Admission Probability: Distribution



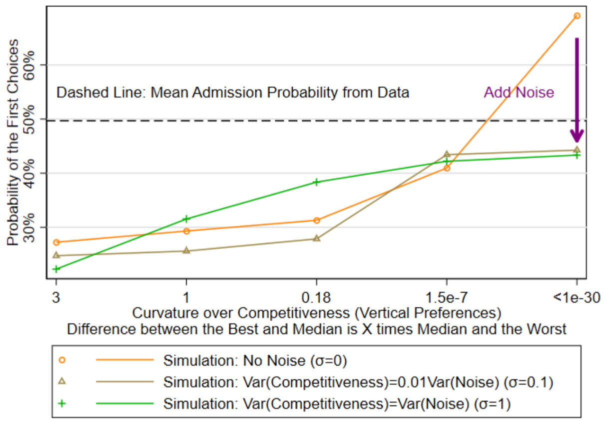
(c) Share of Risk-Taking Reversals



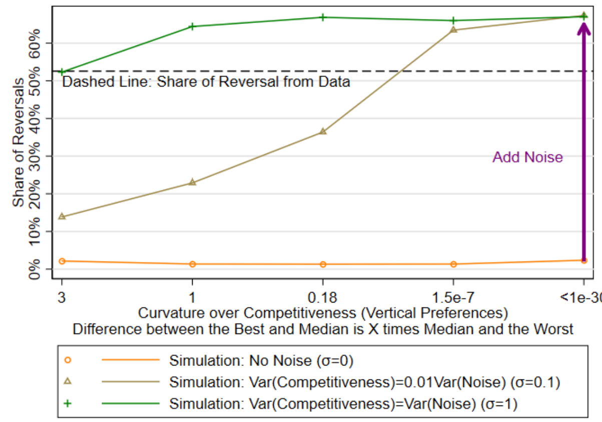
Note: The figure presents summary statistics of unconditional admission probability that are related to the prediction of optimal decision rule among students whose priority scores are top 60%, for the discussion in Section 4.1. Subfigure (a) is a box plot that presents the mean of the estimated unconditional admission probability for the four choices. Subfigure (b) is a box plot that exhibits the distribution of the estimated probability for the first, second, third and fourth choices on the ROL, respectively. The white dashed line shows the median of the distribution, and the upper and lower boundary of the solid box show the 75th and 25th percentile of the distribution, respectively. Subfigure (c) plots the share of students whose strategy exhibits at least one pair of risk-taking reversals anywhere on her ROL, as a function of the threshold above which the reversal is counted in the statistic. The statistic we use to classify reversal is $R \equiv max\{p_i - p_j | \text{for any} 1 \leq j < i \leq 4\}$, only those whose R is above threshold $X\%$ will be counted.

Figure 4: Simulation vs. Data: Cautiousness, Reversals, and Correlation of Cutoffs

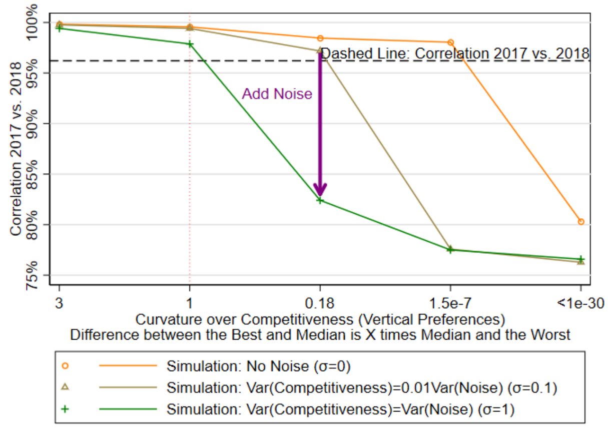
(a) Mean Probability of First Choices



(b) Share of Reversals



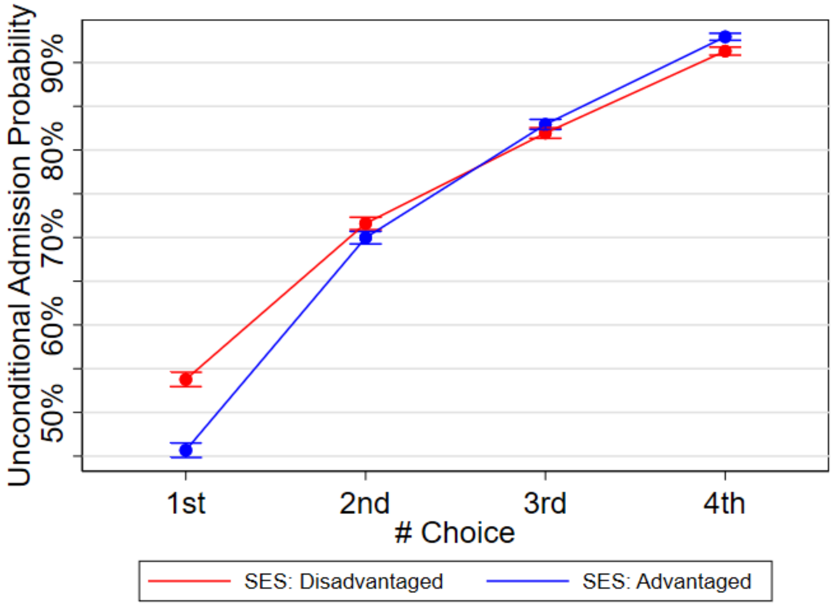
(c) Correlation of Cutoffs between 2017 and 2018



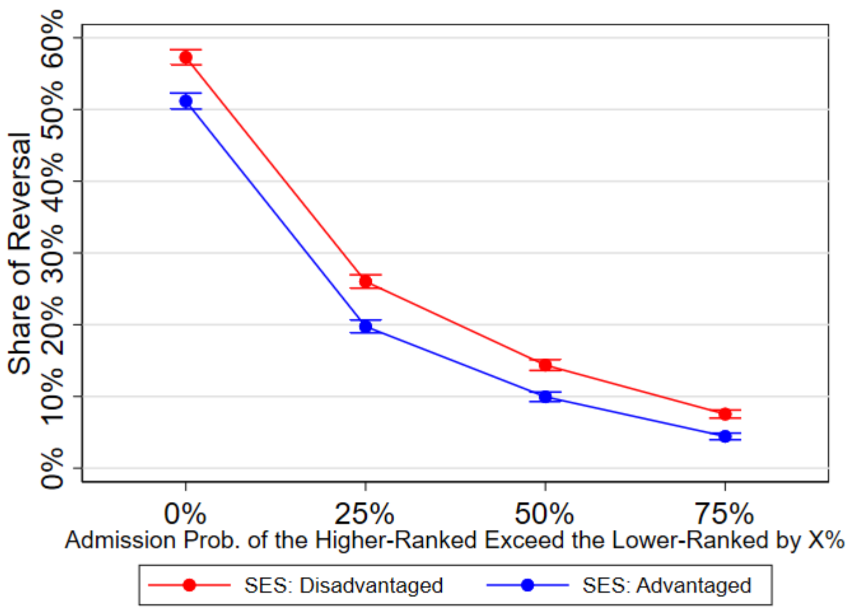
Note: This figure compares the value of empirical and simulated moments along the dimension of first choices and risk-taking reversals among students whose priority scores are top 60%, for the discussion in Section 4.2. In subfigure (a), the vertical axis is the average of unconditional admission probability of first choices. In subfigure (b), the vertical axis is the share of students whose ROL has a reversal pair of which the unconditional probability of the higher-ranked is larger than the lower-ranked. In subfiure (c), the vertical axis is the correlation of cutoffs in 2017 and 2018 for all colleges. The horizontal axis in all subfigures is the curvature over college competitiveness as measured by average normalized cutoffs during 2014-2018, which is controlled by ρ. Solid line and dashed line in both figures represent simulated moments and data, respectively. For simulated moments, we simulate each applicant's choices 50 times and take the average over the whole relevant sample. Hollow circles in orange represent the simulated moments where no noise is added ($\sigma = 0$). Hollow triangles in brown represent the simulated moment where small scale is noise is added ($\sigma = 0.1$). Plus signs in green represent the simulated moments where substantial noise is added ($\sigma = 1$)

Figure 5: Mean of Admission Probability and Share of Reversals by SES Groups

(a) Mean of Unconditional Admission Probability



(b) Share of Risk-Taking Reversals by Threshold X%



Note: This figure presents statistics for the most advantaged quartile (in blue) and the most disadvantaged quartile (in red), respectively. The sample is restricted to those whose priority score is top 60%, for the discussion in Section 4.3. Subfigure (a) plots the mean of unconditional admission probability. Subfigure (b) plots the share of students whose strategy exhibits at least one pair of risk-taking reversals anywhere on her ROL, as a function of the threshold above which the reversal is counted in the statistic. The statistic we use to classify reversal is $R \equiv max\{p_i - p_j | \text{for any} 1 \leq j < i \leq 4\}$, only those whose R is above threshold $X\%$ will be counted. To maximize comparison, the mean from the least advantaged quartile has been reweighted to account for differences in priority scores.

Figure 6: Distribution of Response to Incentivized Survey Questions

ROL 1st Choice: Payoff of non-first colleges is 20 CNY. Indifferent Between (25 CNY, 50%) (R CNY, 25%)			Equivalent Lottery: Indifferent Between (25 CNY, 50%; 20 CNY, 50%) (L CNY, 25%; 20 CNY 75%)			
			Very Cautious L>40	Somewhat Cautious 30<L≤40	Risk Tolerant L≤30	Total
	Very Cautious	R>40	281 19.9%	631 44.7%	62 4.4%	974 69.0%
	Somewhat Cautious	30<R≤40	20 1.4%	289 20.5%	39 2.8%	348 24.7%
	Risk Tolerant	R≤30	3 0.2%	48 3.4%	39 2.8%	90 6.4%
	Total		304 21.5%	968 68.6%	140 9.9%	1412 100%

Note: This figure displays a coarsened joint distribution of responses to Question Group 1 (ROL 1st Choice) and Question Group 2 (Equivalent Lottery), for the discussion in Section 6.2.

Blue cells indicate that students' responses are roughly consistent in the two questions, as predicted by the rational benchmark.

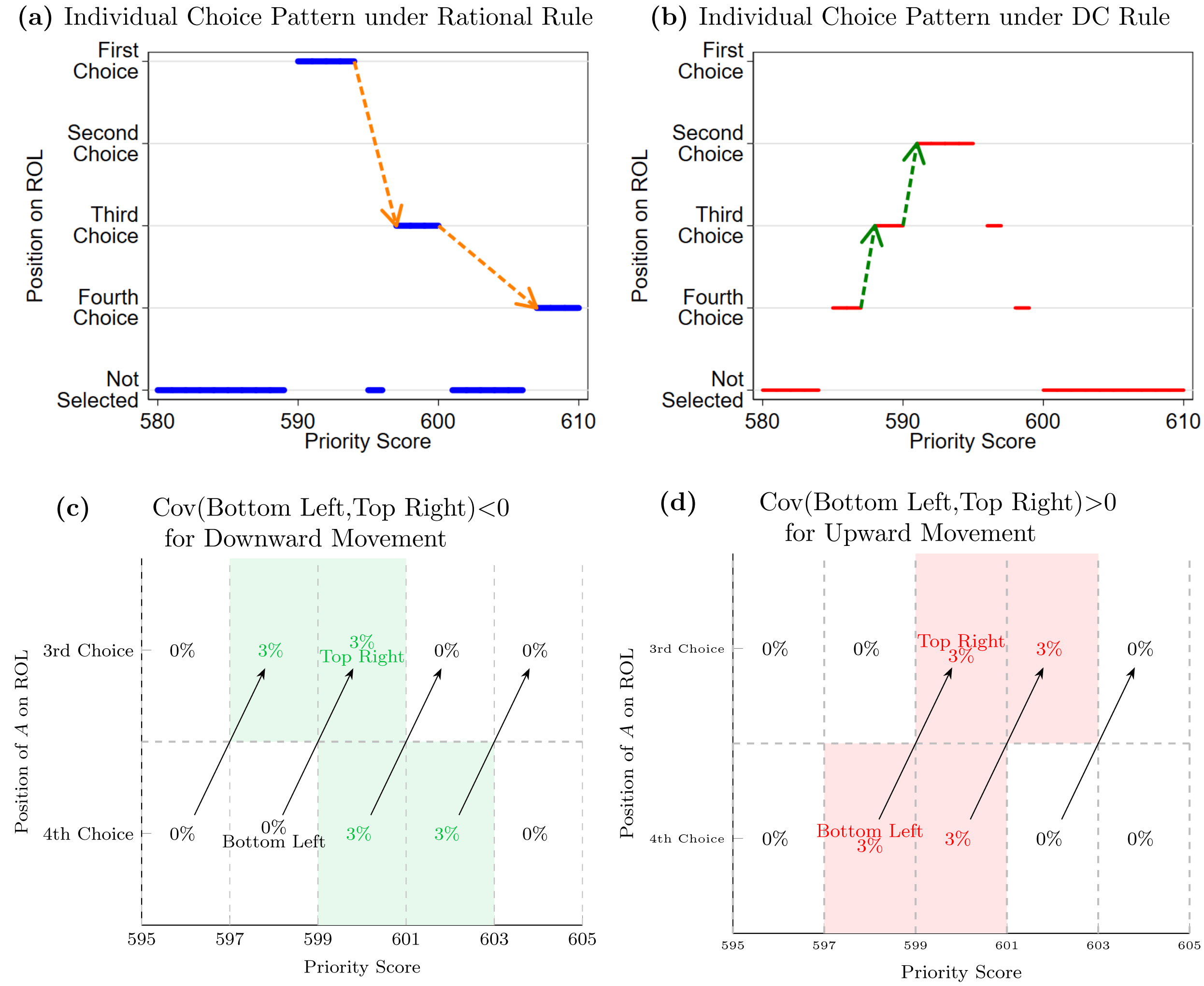
Red cells indicate that students' responses are more cautious in "ROL first-choice" choice, as predicted by the DC decision rule.

Gray cells indicate that students' responses are less cautious in "ROL first-choice" choice, which cannot be predicted by neither decision rule.

"ROL first-choice" question (vertical) corresponds to Question Group 1. In this question, students need to choose from college X, whose admission probability is 50% and payoff of admission is 25 CNY, or college Y, whose admission probability is 25% and payoff of admission is R CNY, where R = 30, 35, 40, 45, 50, 55, 60, respectively in the MPL, and put the college of their choice on the top of their list. If students are not admitted to their first choices, they will be admitted to one of the bottom choices in this scenario, which corresponds to the payoff of 20 CNY.

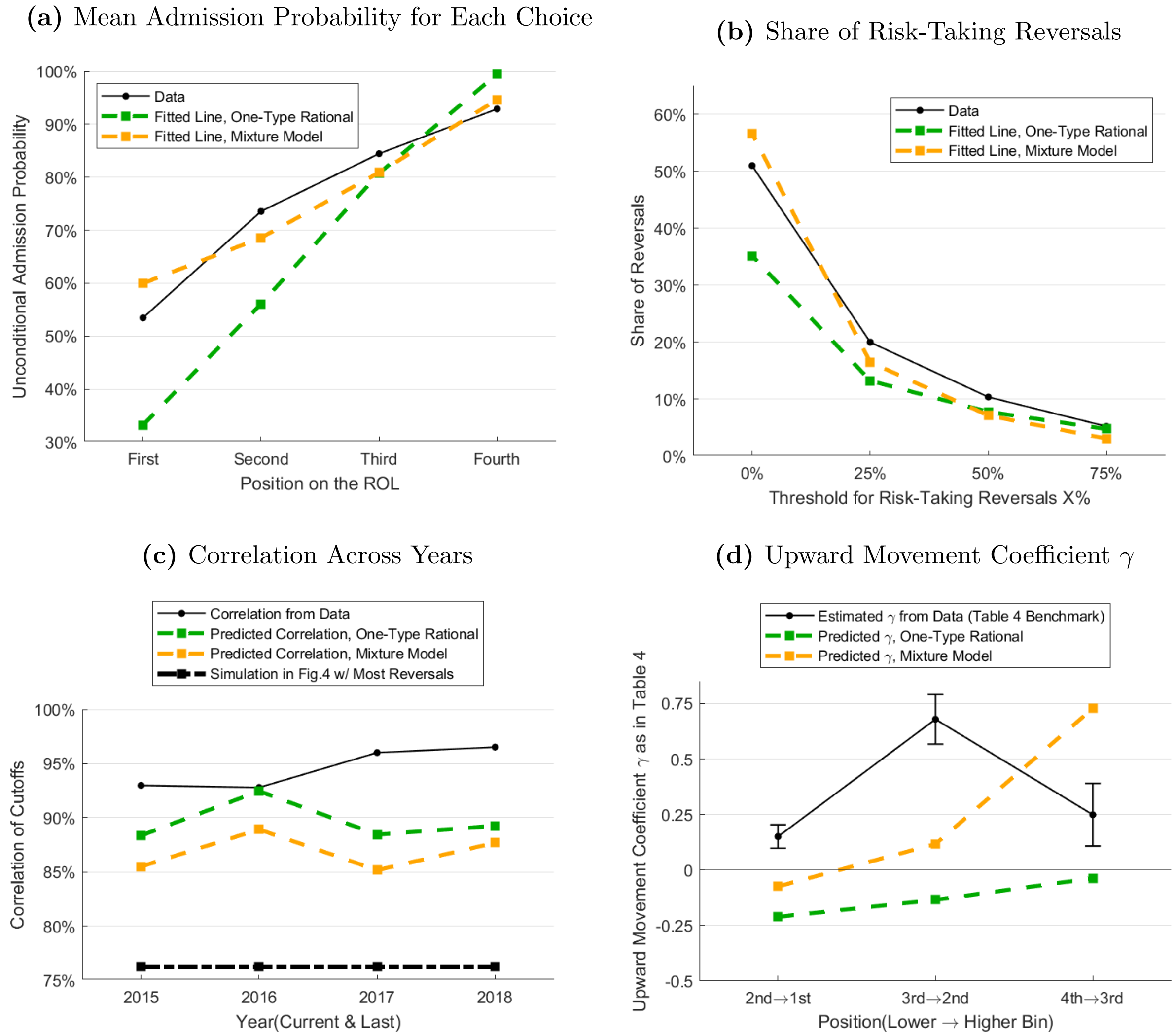
"Equivalent Lottery" question (horizontal) corresponds to Question Group 2. In this question, students need to choose from lottery X, whose payoff is 25 CNY with 50% of chance, 20 CNY with 50% of chance, or lottery Y, whose payoff is L CNY with 25% of chance, 20 CNY with 75% of chance, where L = 30, 35, 40, 45, 50, 55, 60, respectively. Note that "Equivalent Lottery Question" is mathematically equivalent to the aforementioned "ROL first-choice Question".

Figure 7: Prediction: Position Movement in ROL as Priority Score Changes



Note: This figure shows the prediction about how any single college move on the list as priority score changes under Rational Decision Rule and DC Decision Rule respectively, as discussed in Section 5.4, 7.1, and 7.2. Subfigure (a) presents an example where the college in question is put on the list under Rational Decision Rule. Blue dots represent the position of the college given a level of priority score specified on horizontal axis. Subfigure (b) presents an example where the college in question is put on the list under DC Decision Rule. Red dots represent the position of the college given a level of priority score specified on horizontal axis. Subfigure (c) and (d) present an example of share of choice data, where each cell represents the share of students who belong to that score bin who choose to list a college at a specific position. The shares that belong to the same score bin add up to 100%. Subfigure (c) shows that downward movements in general generate negative correlation between bottom left and its neighboring top right cells. Subfigure (d) shows that upward movements in general generate positive correlation between bottom left and its neighboring top right cells.

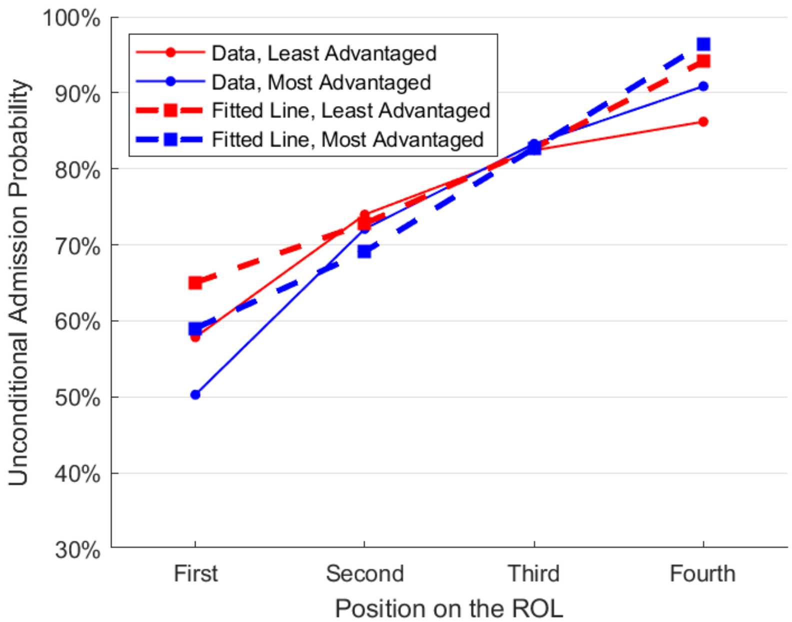
Figure 8: Data & Model Prediction: One-Type Rational vs. Mixture Model



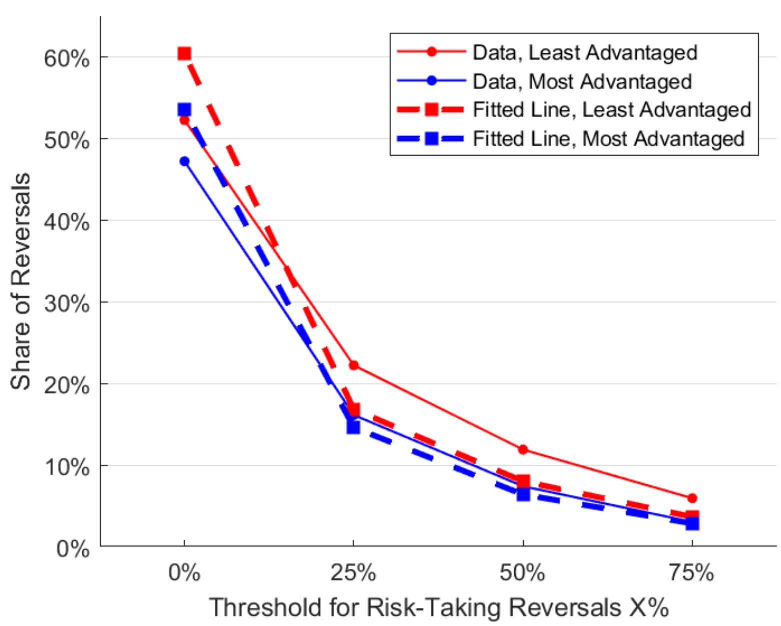
Note: This figure compares the fit and data for the key moments in risk-taking strategies, as discussed in detail in Section 8.3. The fit is generated by the structural model that excludes students of the bottom 40% to minimize the impact of the constraint of priority score on college choices in the year of 2015, 2017, and 2018, where township level SES index can be obtained. Green dashed line is generated by the model that excludes the DC Type, but with the same degree of flexibility in preferences. Orange line is generated by our preferred model, the mixture model that allows for both Rational Type and the DC Type. Subfigure (a) plots the mean of unconditional admission probability for the first, second, third and fourth choices. Subfigure (b) plots the share of risk-taking reversals where the unconditional probability of the higher-ranked exceeds that of lower-ranked by more than X%. Subfigure (c) plots the correlation of simulated cutoffs across years. For example, 2015 means that the statistics concerns the correlation between 2014 cutoffs and 2015 cutoffs. Figure (d) compares the estimated upward movement index from equation 3 and Table 4 to the γ we obtain by doing the same regression on simulated data.

Figure 9: Data & Fit of Mixture Model: Advantaged vs. Disadvantaged

(a) Data vs. Fit: Mean Admission Probability for Each Choice

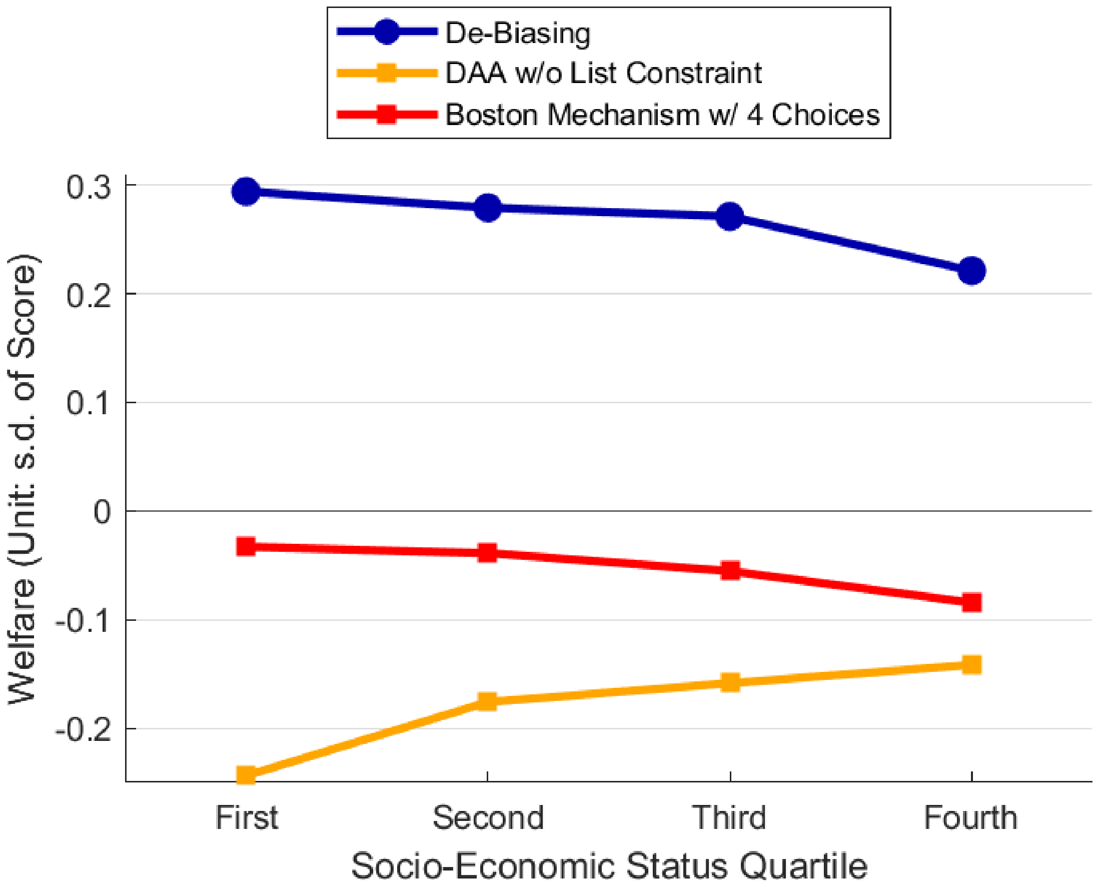


(b) Data vs. Fit: Share of Risk-Taking Reversals



Note: This figure compares the fit and data for the key moments in risk-taking strategies, for the most advantaged quartile (4th quartile) and the least advantaged quartile (1st quartile) respectively, as discussed in detail in Section 8.3. The fit is generated by the structural model that excludes students of the bottom 40% to minimize the impact of the constraint of priority score on college choices in the year of 2015, 2017, and 2018, where township level SES index can be obtained. To maximize comparability across different SES groups, data have been reweighted to account for differences in priority score. Solid lines represent moments from data. Dashed lines represents values simulated by the estimated parameters from the mixture model. Blue lines represent the most advantaged quartile. Red lines represent the least advantaged quartile. Subfigure (a) plots the mean of unconditional admission probability for the first, second, third and fourth choices. Subfigure (b) plots the share of risk-taking reversals with different levels of threshold X%.

Figure 10: Welfare Impact of De-Biasing and Alternative Mechanisms by SES Quartile



Note: This figure presents the mean of welfare impact of de-biasing, as well as switching to alternative mechanisms (Deferred Acceptance Algorithm without list constraints and Boston Mechanism with 4 choices), evaluated separately for students in each quartile of socioeconomic status index. The first quartile is the least advantaged. The fourth quartile is the most advantaged. The discussion of this figure is detailed in Section 8.4.

Table 1: Cautiousness of First and Fourth Choices:
Most Advantaged Quartile vs. Most Disadvantaged Quartile

Unconditional Admission Probability	(1) First	(2) Fourth	(3) Fourth-First
Panel A: Estimated Coefficient			
Most Disadvantaged Quartile	6.74%***	-2.24%***	-8.98%***
	(0.70%)	(0.40%)	(0.78%)
Benchmark Group		Most Advantaged Quartile	
Predicted Mean: Most Advantaged	44.58%	92.11%	47.53%
	(0.43%)	(0.24%)	(0.48%)
Panel B: Predicted Mean of Disadv-Adv Gap, by Quantile of Priority Score			
E[Adv-Disadv|Priority Score 40%]	5.77%	-1.66%	-7.43%
	(0.96%)	(0.54%)	(1.08%)
E[Adv-Disadv|Priority Score 60%]	7.72%	-1.97%	-9.69%
	(0.87%)	(0.49%)	(0.97%)
E[Adv-Disadv|Priority Score 80%]	10.36%	-1.88%	-12.23%
	(1.67%)	(0.94%)	(1.87%)
E[Adv-Disadv|Priority Score 99%]	8.76%	-3.72%	-12.48%
	(5.94%)	(3.37%)	(6.65%)
Panel C: List of Controls			
4th-Order Polynomial of Priority Score	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes
Track (Science or Humanity) Fixed Effects	Yes	Yes	Yes

Note: This table reports reduced-form results from administrative data that analyzes the unconditional probability of students' first choices and fourth choices among students from the most disadvantaged quartile and the most advantaged quartile, as described in Section 4.3. All statistics are generated by a fixed effect regression that regresses outcome variables on the interaction of an indicator whether students belong to the most disadvantaged, and a fourth-order polynomial of priority scores. Column 1 reports statistics related to the unconditional probability of the first choices. Column 2 reports statistics related to the unconditional probability of the fourth choices. Column 3 reports statistics related to the fourth-first choice gap, in terms of unconditional probability. Panel A reports the main effect of belonging to the most disadvantaged. Panel B examines the heterogeneity of adv-disadv gap by reporting the predicted mean of gap conditional on priority score quantile. Panel C reports the additional controls. All statistics are in their percentage form * significant at 10%, ** significant at 5%, *** significant at 1%.

Table 2: Risk-Taking Reversals: Most Advantaged Quartile vs. Most Disadvantaged Quartile

	(1)	(2)	(3)	(4)
Threshold of Reversal X%: Probability of the Higher-Ranked Exceeds That of the Lower-Ranked by More Than X%	0%	25%	50%	75%
Panel A: Estimated Coefficient				
Most Disadvantaged Quartile	5.55%***	6.64%***	4.97%***	3.91%***
	(0.94%)	(0.80%)	(0.64%)	(0.48%)
Benchmark Group	Most Advantaged Quartile			
Predicted Mean of Benchmark Group	53.12%	21.43%	11.13%	4.94%
	(0.57%)	(0.49%)	(0.39%)	(0.29%)
Panel B: Predicted Mean of Disadv-Adv Gap, by Quantile of Priority Score				
E[Adv-Disadv|Priority Score 40%]	6.71%	7.10%	6.00%	5.79%
	(1.28%)	(1.10%)	(0.87%)	(0.65%)
E[Adv-Disadv|Priority Score 60%]	4.87%	5.89%	4.24%	3.05%
	(1.16%)	(0.99%)	(0.79%)	(0.59%)
E[Adv-Disadv|Priority Score 80%]	3.36%	4.31%	2.23%	0.27%
	(2.23%)	(1.90%)	(1.51%)	(1.13%)
E[Adv-Disadv|Priority Score 99%]	9.78%	8.40%	4.31%	1.58%
	(7.94%)	(6.79%)	(5.4%)	(4.04%)
Panel C: List of Controls				
4th-Order Polynomial of Priority Score	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Track (Science or Humanity) Fixed Effects	Yes	Yes	Yes	Yes

Note: This table reports reduced-form results from administrative data that analyzes the risk-taking reversal, namely, choosing rank a safer college higher, among students from the most disadvantaged quartile and the most advantaged quartile, as described in Section 4.3. We classify a pair of choices as risk-taking reversal if the gap in terms of admission probability exceeds X%, where X takes the value of 0, 25, 50, 75 in Columns 1, 2, 3, 4, respectively. All statistics are generated by a fixed effect regression that regresses outcome variables on the interaction of an indicator whether students belong to the most disadvantaged, and a fourth-order polynomial of priority scores. Panel A reports the main effect of belonging to the most disadvantaged. Panel B examines the heterogeneity of adv-disadv gap by reporting the predicted mean of gap conditional on priority score quantile. Panel C reports the additional controls. All statistics are in their percentage form * significant at 10%, ** significant at 5%, *** significant at 1%.

Table 3: Testing Framing Effect - Incentivized Survey Responses

Panel A: Response Inconsistency Between ROL and Lottery Question			
	(1)	(2)	(3)
	ROL 1st Choice: Payoff of non-1st colleges is 20 CNY. Indifferent Between (25 CNY, 50%) &(R CNY, 25%) Equivalent Lottery: Indifferent Between (25 CNY, 50%; 20 CNY, 50%) & (Y CNY, 25%; 20 CNY 75%) Inconsistency Measure: |R-Y|		
SES Index Normalized	0.884***	0.817***	0.804***
	(0.226)	(0.234)	(0.286)
Mean of Inconsistency		10.166	
		(0.313)	
Control: Priority Score	Yes	Yes	Yes
Control: Demographic Variables	No	Yes	Yes
Control: College Preferences	No	No	Yes
R squared	0.023	0.032	0.093
Observations	1412	1412	1412
Panel B: Estimated Share of Type and Risk Preferences from Survey Responses			
	(1)	(2)	(3)
Share of Directed Cognition Type: Mean	0.0%	49.8%	48.7%
	-	(1.8%)	(1.8%)
Marginal Effect of Normalized SES	0.0%	-7.0%	-7.3%
	-	(1.9%)	(1.7%)
Marginal Effect of Normalized Score	0.0%	-3.8%	-4.0%
	-	(1.5%)	(1.5%)
Share of Sincere Type	0.0%	0.0%	3.1%
	-	-	(0.5%)
CRRA Risk Coefficient: Mean	0.239	0.056	0.079
	(0.013)	(0.014)	(0.013)
CRRA Risk Coefficient: SES	-0.013	0.010	0.010
	(0.012)	(0.015)	(0.013)
CRRA Risk Coefficient: Normalized Score	-0.016	-0.009	-0.011
	(0.013)	(0.012)	(0.011)
Number of Parameters	6	9	10
Log Likelihood	-6271.152	-5770.302	-5733.326
Number of Observations	1412	1412	1412
Bayesian Information Criterion	12585.82	11605.88	11539.18

Note: This table reports empirical analysis from incentivized survey response, as described in Section 6.2. Panel A analyzes to what extent the inconsistency of risk-taking across "ROL 1st Choice" question and "Equivalent Lottery" question is correlated with students' socioeconomic status and their priority scores, controlling for other variables elicited in the survey. Panel B jointly estimates students' propensity of being a Directed-Cognition Type and their CRRA risk preferences using different specifications. Column (1) excludes any behavioral type. Column (2) estimates a mixture model of the DC and the rational type. Column (3) estimates a model that additionally allows for sincere type as theorized in Pathak and Sönmez (2008).

Table 4: Testing Upward Movement - Choice Share at Lower Positions (Bottom Left Cell) Correlates with Choice Share at Higher Positions in Higher Adjacent Bin (Top Right Cell)

Variable Names	Panel A: Baseline Regression			Panel B: Robustness (Bin Size 25% Smaller)		
	(1)	(2)	(3)	(1)	(2)	(3)
	Share of Choice in Bin s as X th Choice					
	1st	2nd	3rd	1st	2nd	3rd
Share of Choice in Bin (s-1) as 2nd Choice	0.151***			0.271***		
	(0.027)			(0.026)		
Share of Choice in Bin (s-1) as 3rd Choice		0.679***			0.414***	
		(0.057)			(0.054)	
Share of Choice in Bin (s-1) as 4th Choice			0.249***			0.103***
			(0.072)			(0.038)
College Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Score Bin Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes

Variable Names	Panel C: Robustness (All bins 1 point higher)			Panel D: Robustness (All bins 1 point lower)		
	(1)	(2)	(3)	(1)	(2)	(3)
	Share of Choice in Bin s as X th Choice					
	1st	2nd	3rd	1st	2nd	3rd
Share of Choice in Bin (s-1) as 2nd Choice	0.309***			0.308***		
	(0.026)			(0.033)		
Share of Choice in Bin (s-1) as 3rd Choice		0.171***			0.387***	
		(0.036)			(0.042)	
Share of Choice in Bin (s-1) as 4th Choice			0.213***			0.297***
			(0.063)			(0.074)
College Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Score Bin Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes

Note: This table corresponds to the empirical analysis in Section 7.2. It reports regression results using specification 3. All columns concern whether choice share for a position at lower bin correlates choice share for a higher position at higher adjacent bin. Substantial presence of DC type generates upward movement that leads to correlation of variables of interest, whereas a sample that only consists of the rational type generates no or negative correlation. In each panel, Columns (1), (2), (3) present whether choice share for 2nd, 3rd, 4th spot predicts choice share for 1st, 2nd, 3rd at higher adjacent bin, respectively. Both college fixed effects and score bin fixed effects are included in all columns. Panel A presents estimation results using the baseline way to construct score bins. Panel B presents results when the length of all bins is 25% shorter than the baseline. Panel C presents results when all bins are shifted 1 point lower. Panel D presents results when all bins are shifted 1 point higher. Standard errors are reported in parentheses. * significant at 10%, ** significant at 5%, *** significant at 1%.

Table 5: Structural Estimation of Mixture Model using Administrative Data

Panel A: Estimation Results - Rational One-Type vs. Mixture Model						
	(1)	(2)	(3)	(4)	(5)	(6)
Sample (Quantile of Priority Score)	40%-60%		60%-80%		80%-100%	
Model	Rational	Mixture	Rational	Mixture	Rational	Mixture
Estimated Share of DC Type		53.1%		45.1%		55.1%
		(0.61%)		(0.54%)		(0.55%)
Marginal Effect of SES		-3.71%		-3.68%		-6.02%
		(0.57%)		(0.56%)		(0.55%)
Mean Curvature: Rational Type	-0.044	-0.375	0.413	-0.365	0.307	-0.336
	(0.007)	(0.027)	(0.022)	(0.024)	(0.014)	(0.021)
Mean Curvature: DC Type		-0.486		-0.459		-0.478
		(0.055)		(0.033)		(0.021)
Number of Moments	120	120	120	120	120	120
Number of Parameters	20	29	20	29	20	29
Distance	7699.344	4462.128	7008.675	2166.753	4737.862	2213.298
Decrease (Improvement) of Distance in Percentage	42.0%		69.1%		53.3%	
MMSC-BIC (Andrews&Lu, 2001)	6864.396	3702.324	6174.988	1408.098	3903.911	1454.404
Panel B: Out-of-Sample Predictions - Rational One-Type vs. Mixture Model						
Sample (Quantile of Priority Score)	40%-60%		60%-80%		80%-100%	
Model	Rational	Mixture	Rational	Mixture	Rational	Mixture
Distance	5760.837	4903.896	5477.374	3102.324	3947.603	2083.348
Decrease (Improvement) of Distance in Percentage	14.9%		43.4%		47.2%	
Key Moments						
Data: Mean Probability of 1st Choices	44.1%		45.6%		63.6%	
Prediction: Mean Probability of 1st Choices	29.0%	58.4%	32.3%	55.5%	38.5%	67.3%
Data: Mean Probability of 4th Choices	85.8%		87.8%		95.0%	
Prediction: Mean Probability of 4th Choices	99.8%	94.3%	99.3%	92.0%	99.4%	97.4%
Data: Share of Reversals	61.3%		58.0%		51.9%	
Prediction: Share of Reversals	33.0%	52.3%	37.1%	62.2%	36.1%	56.7%
Panel C: Welfare Evaluation Using Mixture Model (Unit: Standard Deviation of Priority Score)						
Sample (Quantile of Priority Score)	40%-60%		60%-80%		80%-100%	
Type	Rational	DC	Rational	DC	Rational	DC
Debiasing	0.368	0.495	0.217	0.253	-0.080	0.082
Deferred Acceptance with Unrestricted List	-0.329	0.231	-0.725	0.447	-0.045	-0.155
Boston Mechanism with 4 Choices	-0.147	-0.066	-0.074	-0.021	-0.152	-0.053

Note: This table reports the results of structural analysis as described in Sections 8.3 and 8.4, for the 40% $\sim$ 60% (column 1,2), 60% $\sim$ 80% (column 3,4), and 80% $\sim$ 100% (column 5,6) subsample. Columns 2,4,6 report estimation of mixture model where DC and Rational Type coexist. Columns 1,3,5 report estimation results of a rational model with flexible structure in college preferences. Standard errors are in parentheses. MMSC-BIC is a model and moments selection criteria for GMM developed by Andrews and Lu (2001). It is analogous to Bayesian Information Criterion in the context of maximum likelihood estimation.

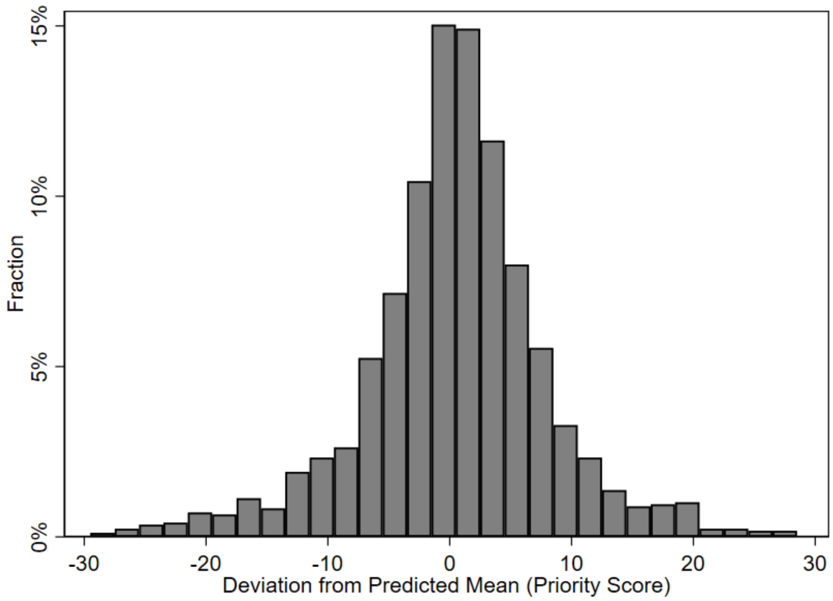
Table 6: Alternative Specifications of Structural Estimation

Panel A: Alternative Specifications - Less Flexible Preferences						
	(1)	(2)	(3)	(4)	(5)	(6)
Sample (Quantile of Priority Score)	40%-60%		60%-80%		80%-100%	
	Rational	Mixture	Rational	Mixture	Rational	Mixture
Estimated Share of DC Type		49.3%		47.8%		54.0%
		(0.25%)		(0.23%)		(0.30%)
Marginal Effect of SES		0.99%		-2.31%		-5.03%
		(0.33%)		(0.26%)		(0.31%)
Mean Curvature: Rational Type	-0.296	-0.312	-0.133	-0.500	0.259	-0.500
	(0.015)	(0.019)	(0.011)	(0.021)	(0.013)	(0.017)
Number of Moments	120	120	120	120	120	120
Number of Parameters	8	11	8	11	8	11
Distance	11647.42	4586.36	9181.853	4171.124	7247.197	3102.797
Decrease of Distance in Percentage	60.6%		54.6%		57.2%	
MMSC-BIC (Andrew&Lu, 2001)	10712.27	3676.267	8246.711	3261.031	6312.054	2192.703
Panel B: Alternative Specifications - Heterogeneous Beliefs						
Sample (Quantile of Priority Score)	40%-60%		60%-80%		80%-100%	
	Rational	Mixture	Rational	Mixture	Rational	Mixture
Estimated Share of DC Type		92.3%		96.2%		97.8%
		(0.51%)		(0.28%)		(0.26%)
Marginal Effect of SES		6.45%		-1.00%		-0.97%
		(0.48%)		(0.33%)		(0.40%)
Mean Curvature: Rational Type	-0.318	0.125	-0.100	4.529	0.090	10.445
	(0.016)	(0.045)	(0.011)	(1.171)	(0.005)	(9.113)
Mean Curvature: DC Type		-0.494		-0.478		-0.431
		(0.022)		(0.017)		(0.011)
Number of Moments	120	120	120	120	120	120
Number of Parameters	20	29	20	29	20	29
Distance	10026.09	4586.656	30492.2	4331.319	54968.45	3360.928
Decrease of Distance in Percentage	54.3%		85.8%		93.9%	
MMSC-BIC (Andrew&Lu, 2001)	9191.145	3826.853	29657.25	3571.516	54133.5	2601.125

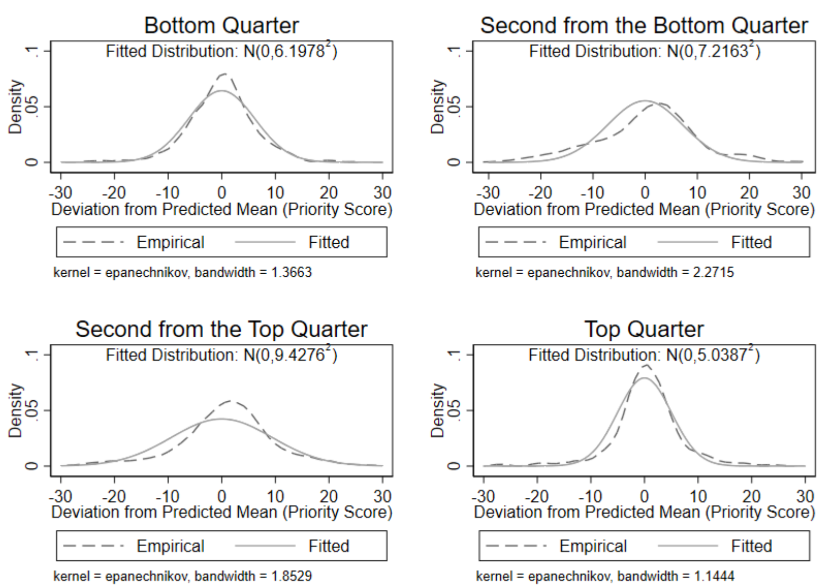
Note: This table reports the results of structural analysis as described in Section 8.3 and 9.1, for the 40% $\sim$ 60% (column 1,2), 60% $\sim$ 80% (column 3,4), and 80% $\sim$ 100% (column 5,6) subsample using alternative specifications. Panel A reports the estimation results where the model is the same as the single-type/mixture model in Table 5 except that only preferences over competitiveness and distance is taken into account, and is homogeneous across types. Panel B reports the estimation results where the parameters are the same as in Table 5 but we additionally introduce perturbation in beliefs with the parameters of the perturbation distribution calibrated using data from the online survey.

A Supplementary Figures

Figure A1: Distribution of Realized Cutoff-Predicted Mean Differences



(a) Distribution of Differences: All Colleges

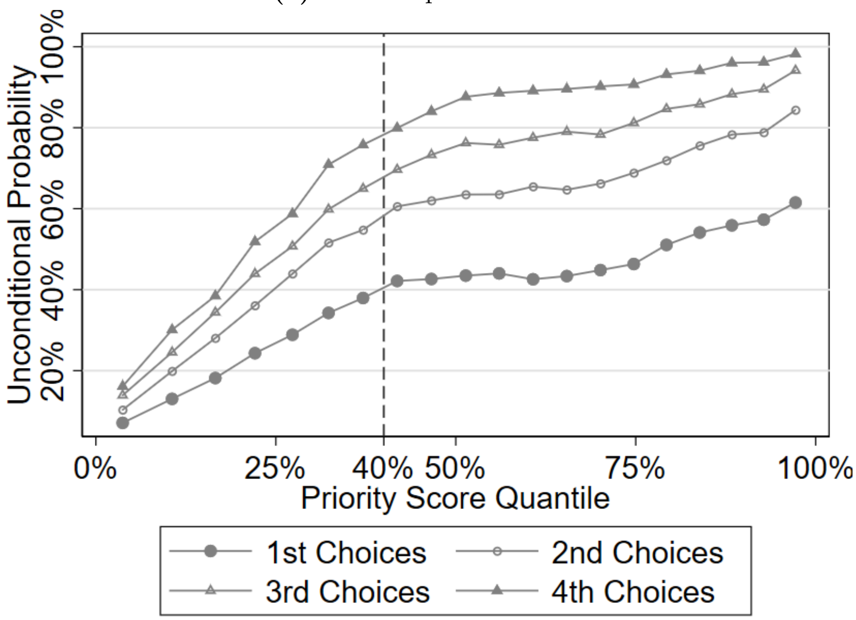


(b) Distribution of Differences by College Competitiveness

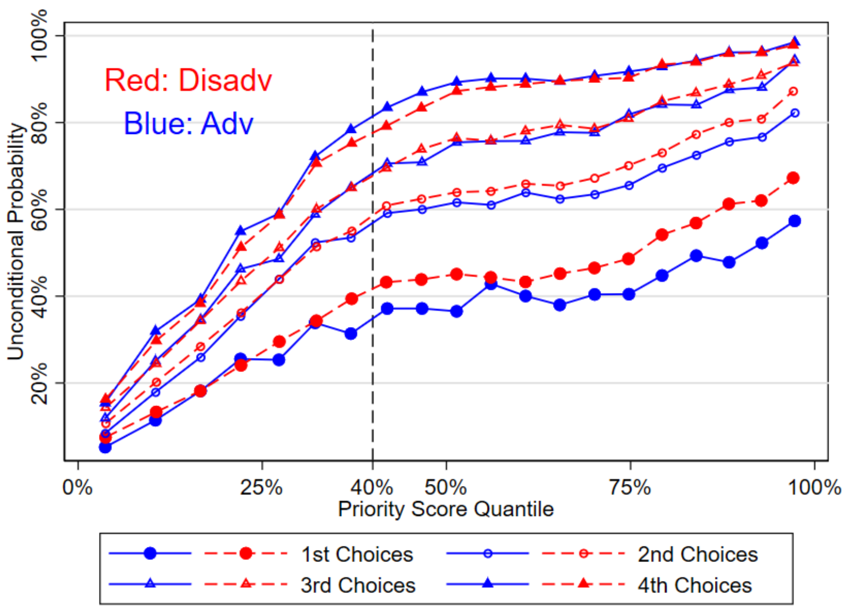
Note: This figure plots the distribution of distance between realized cutoffs and predicted mean as defined in Section 3.2. Subfigure (a) plots the distribution of differences between admission cutoffs and predicted mean among all colleges. Subfigure (b) plots the distribution of differences by college competitiveness, where the dashed lines in the top left, top right, bottom left, and bottom right graph are the empirical distribution for the least competitive quarter of colleges, second from the least competitive quarter, second from the most competitive quarter, and the most competitive quarter of colleges, respectively. The solid lines in subfigure (b) are fitted distributions using the median estimate of the standard deviation of cutoff-mean difference for corresponding quarter of colleges. Both subfigures omit outliers which is more than 30 points away from the predicted mean.

Figure A2: Mean of Admission Probability Conditional on Priority Score

(a) Full Sample

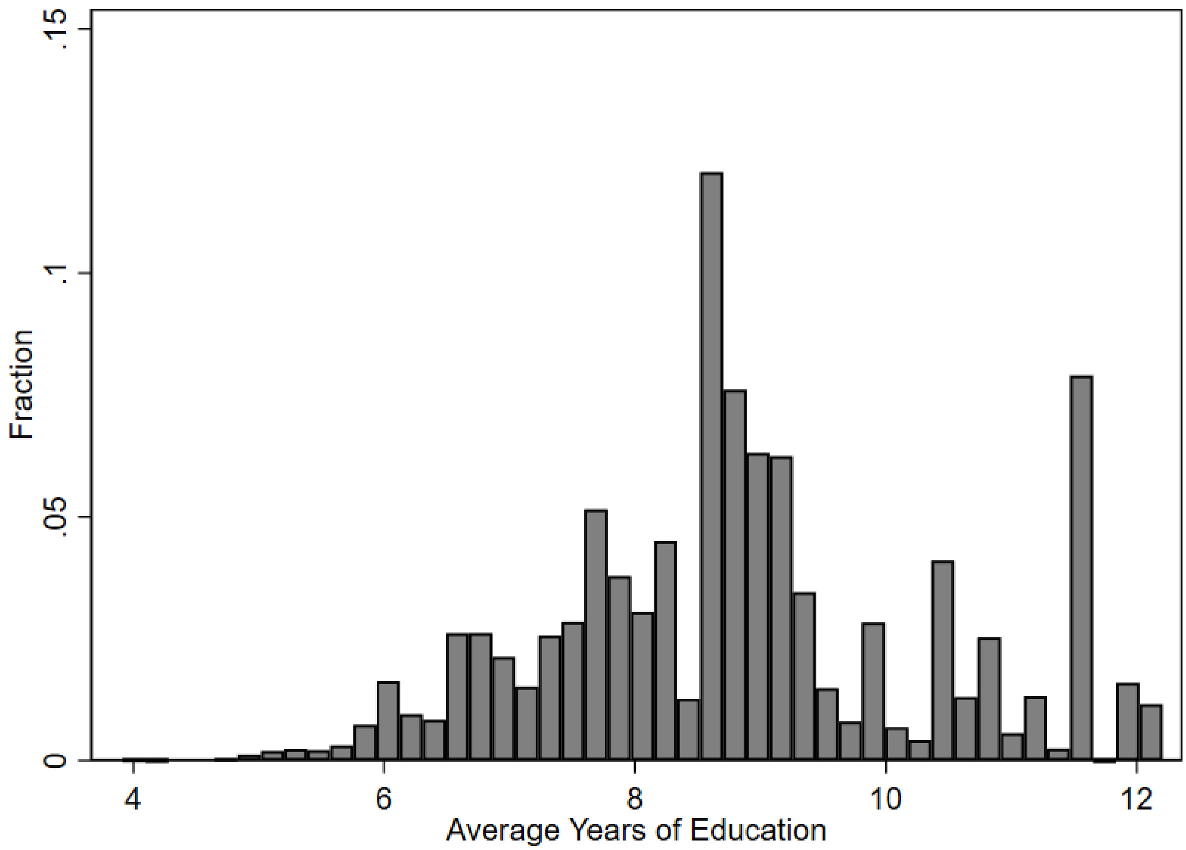


(b) Advantaged cs. Disadvantaged



Note: This figure plots the mean of admission probability conditional on the quantile of priority score for student applicants' first choices, second choices, third choices and fourth choices, respectively. Subfigure (a) plots the statistics for the entire sample. Subfigure (b) plots the statistics for the advantaged students (the most advantaged quartile in terms of township-level education attainment) in blue, and for the other students in red. This graph is helpful for the discussion in Section 4.

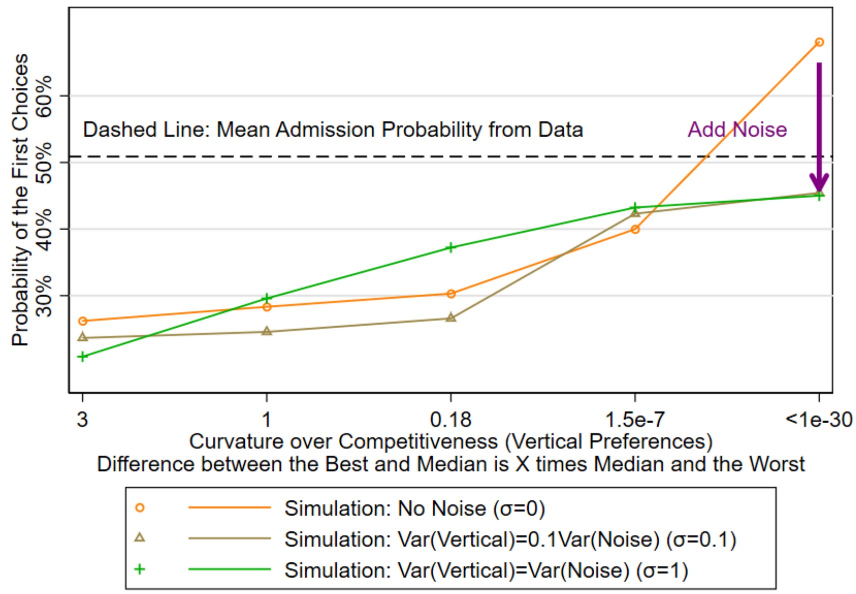
Figure A3: Average Years of Education in Administrative Data (Township Level)



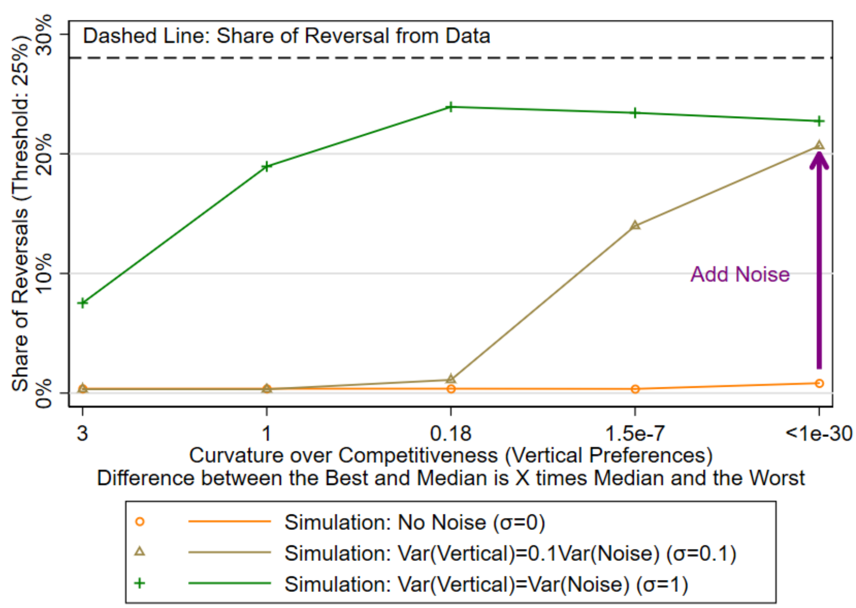
Note: This figure plots the cumulative distribution of probability of meeting cutoffs for students' first (in red), second (in yellow), third (in green) and fourth (in blue) choices, respectively, during 2014-2018. This graph is helpful for the discussion in Section 4.3.

Figure A4: Simulation vs. Data: Probability of First Choices and Share of Reversals
The Most Disadvantaged Quartile

(a) Mean Probability of First Choices

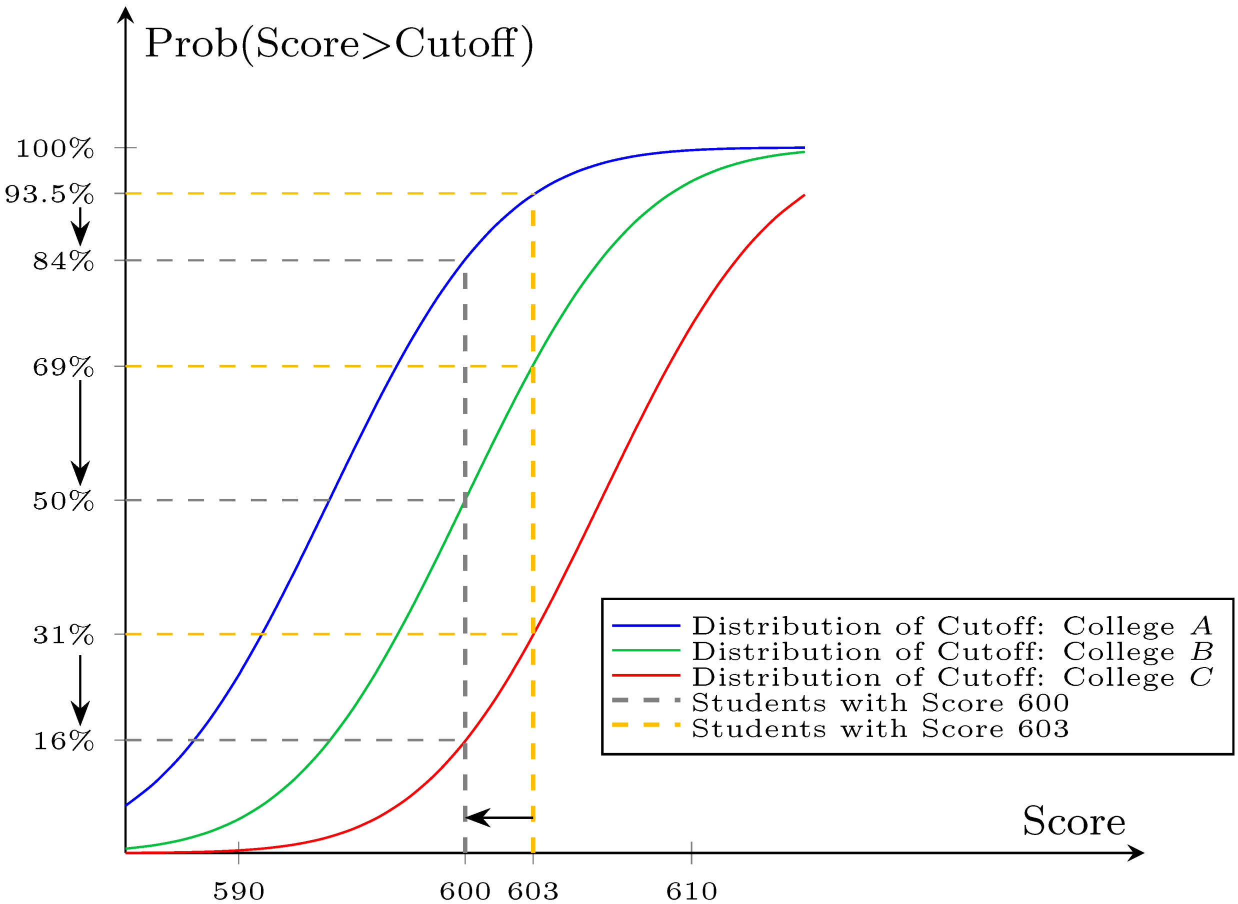


(b) Share of Reversals



Note: This figure compares the value of empirical and simulated moments along the dimension of first choices and risk-taking reversals among students whose priority scores are top 60%, and are from the most disadvantaged quartile. In subfigure (a), the vertical axis is the average of unconditional admission probability of first choices. In subfigure (b), the vertical axis is the share of students whose ROL has a reversal pair of which the unconditional probability of the higher ranked is at least 25% larger than the lower-ranked. The horizontal axis in all subfigures is the curvature over college competitiveness as measured by average normalized cutoffs during 2014-2018, which is controlled by ρ. Solid line and dashed line in both figures represent simulated moments and data, respectively. For simulated moments, we simulate each applicant's choices 50 times and take the average over the whole relevant sample. Hollow circles in orange represent the simulated moments where no noise is added ($\sigma = 0$). Hollow triangles in brown represent the simulated moment where small scale is noise is added ($\sigma = 0.1$). Plus signs in green represent the simulated moments where substantial noise is added ($\sigma = 1$). This graph is helpful for the discussion in Section 4.3.

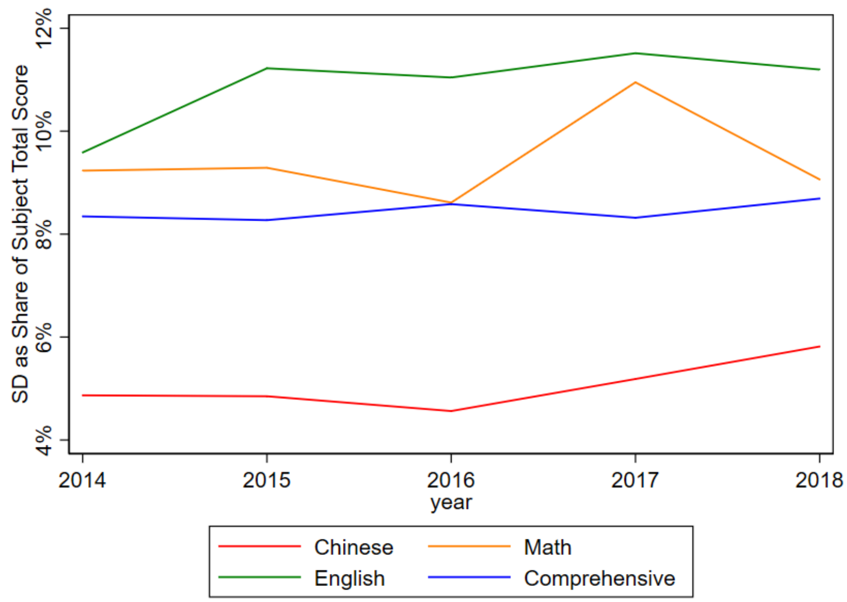
Figure A5: Small decrease in priority score leads to sizable change in admission probability



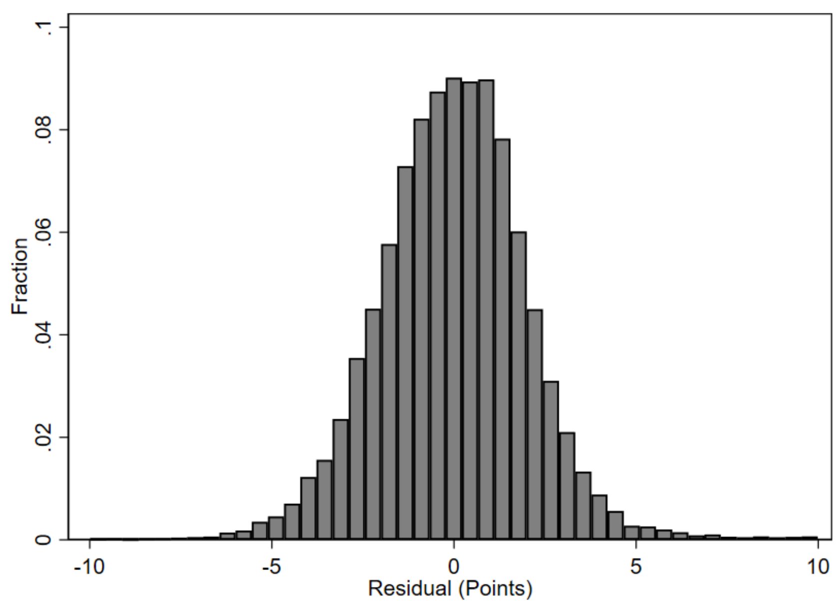
Note: This figure plots an example where a marginal decrease in priority score leads to sizable changes in admission probability. The variance of cutoffs are selected to reflect comparable level of uncertainty that we see in the data. This graph is helpful for the discussion at the beginning of Section 8.

Figure A6: Impact of Subject Difficulty Variation on Admission Probability

(a) Difficulty of Subjects Varies Across Years

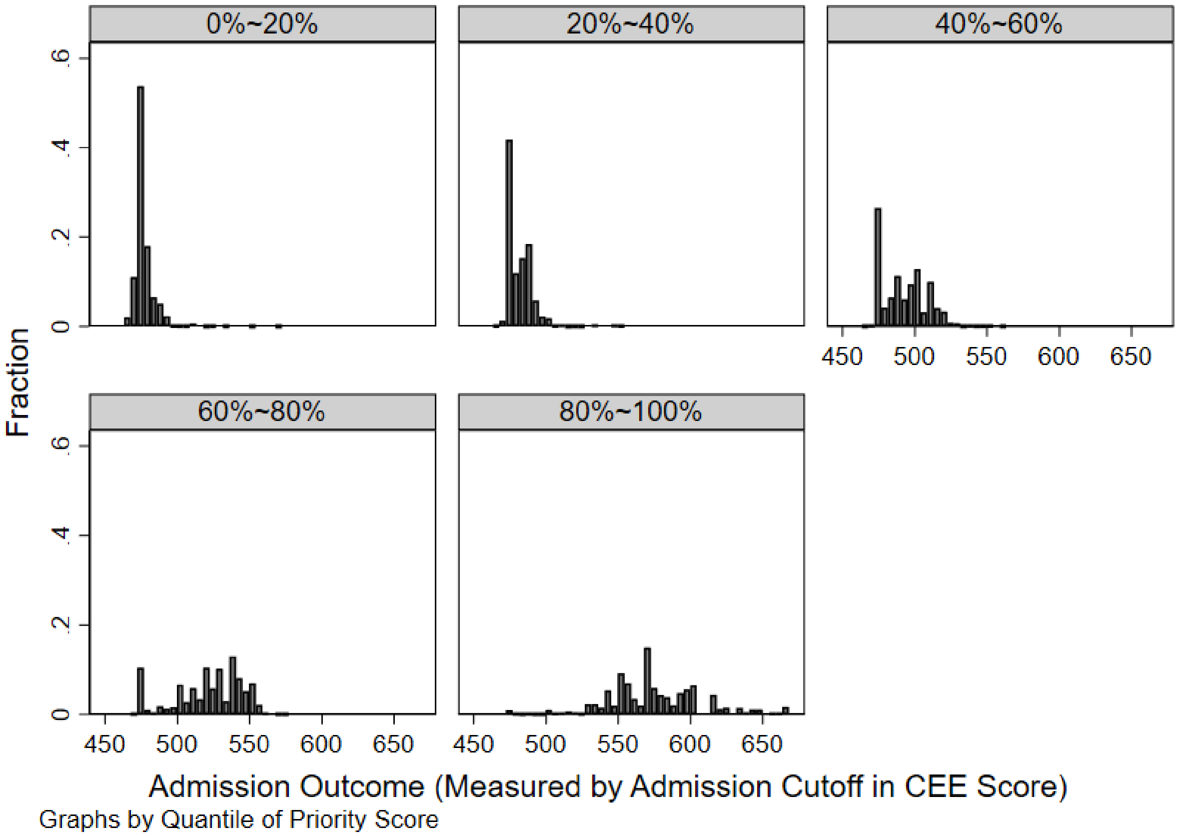


(b) Magnitude of Shock to Admission Probability is Sizable



Note: This figure explains the source of exogenous variation in admission probability. Subfigure (a) presents the standard deviation of raw exam scores as share of subject total scores for each subject during 2014-2018. Subfigure (b) is a histogram of the estimated distribution of shocks, rescaled in terms of the predicted standard deviation of cutoff of students' first choices. This graph is helpful for the discussion in Section 7.1.

Figure A7: Competitiveness of Admitting College: by CEE Score Quantile



Note: This figure plots the distribution of colleges that admit students during 2014-2018. We split the entire sample of STEM applicants into five groups according to their CEE Score, with the first quintile being the group with lowest CEE Score and the fifth being the highest. This graph is helpful for the discussion in Section 8.

B Supplementary Tables

Table B1: Examples of Admission Rules

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Ex. Number	Priority Score	Admission Cutoffs				Admission Outcome
		A	B	C	D	
1	600	595	590	587	580	A
2	580	581	572	583	550	B
3	577	595	590	587	580	None
4	580	595	590	587	580	D

Note: This table presents four examples in which students with different priority scores applying for different sets of colleges in their ROLs. The serial number of examples is in Column (1). Each example is associated with a hypothetical student. The priority scores of the students are recorded in Column (2). Columns (3)-(6) present the admission cutoffs of the colleges that the students put in their ROLs. Column (7) presents the admission outcome of each hypothetical student as a result of their scores and ROLs. This table is helpful for the discussion in Section 2.3.

Table B2: Validity of Estimates of Unconditional Probability

	(1)	(2)	(3)	(4)
	Admitted to First		Admitted to Second	
Estimated Uncond. Prob. to First	1.0015***	1.0117***		
	(0.0043)	(0.0081)		
Estimated Uncond. Prob. to Second			0.9883***	1.0252***
			(0.0060)	(0.0107)
Constant	-0.0043*	-0.0050	0.0033	-0.0094
	(0.0024)	(0.0040)	(0.0037)	(0.0060)
Subsample (Science/Humanity)	Science	Humanity	Science	Humanity
Share of Admitted	.41	.314	.481	.419
Predicted Share of Admitted	.413	.316	.484	.418
F-Test: $\alpha = 0$, $\beta = 1$	.074	.328	.085	.059
Number of Observations	36104	8230	20929	5571
R squared	0.605	0.652	0.566	0.624

Note: This table reports the empirical exercise that tests the validity of probability estimates that we construct in Section 3.2. Columns 1 and 2 present the results of the regression where we regress the outcome of being admitted to the first choices on the estimated probability of meeting the cutoff of first choices, using the full sample of science-track and humanity-track students, respectively. Columns 3 and 4 present the results of the regression where we regress the outcome of being admitted to the second choices on the estimated probability of meeting the cutoff of second choices using the subsample of students who are not admitted to their first choices, for science-track and humanity track, respectively. This table is helpful for the discussion in 3.2.

Table B3: Admission Outcome and Score-Cutoff Gap

	(1)	(2)	(3)
	Selectivity (Normalized)		
Most Disadvantaged Quartile	-0.1046***	-0.0834***	-0.0778***
	(0.0052)	(0.0080)	(0.0138)
Benchmark Group	Most Advantaged Quartile		
CEE Score	Yes	Yes	Yes
Demographic Variables	No	Yes	Yes
County Fixed Effects	No	No	Yes
Number of Observations	20451	20441	20441
R squared	0.894	0.895	0.896

Note: This table compares the selectivity of admitting college, as measured by cutoffs during 2014-2018 (normalized), for the most advantaged quartile to the least advantaged quartiles, with different sets of controls in different columns. Only the most advantaged and most disadvantaged quartile are included in the regression. Students' CEE scores have been controlled in all columns. Demographic variables are added as controls for Columns 2,3,5,6. County Fixed Effects are controlled in Columns 3 and 6. Standard errors are in parentheses. * significant at 10%, ** significant at 5%, *** significant at 1%. This table is helpful for the discussion in Section 4.3.

Table B4: Design of Incentivized Questions in Survey

	Panel A1: ROL for Question Group 1			Panel C1: ROL for Question Group 3		
	ROL #	Payoffs	Prob(Meeting Cutoffs)	ROL #	Payoffs	Prob(Meeting Cutoffs)
	1st	?	?	1st	?	?
	2nd	20 CNY	100%	2nd	5 CNY	100%
	3rd	20 CNY	100%	3rd	5 CNY	100%
	4th	20 CNY	100%	4th	5 CNY	100%

	Panel A2: MPL for Question Group 1		Panel C2: MPL for Question Group 3	
Question #	College X	College Y	College X	College Y
(1)	25 CNY, 50%	30 CNY, 25%	25 CNY, 50%	30 CNY, 25%
(2)	25 CNY, 50%	35 CNY, 25%	25 CNY, 50%	35 CNY, 25%
(3)	25 CNY, 50%	40 CNY, 25%	25 CNY, 50%	40 CNY, 25%
(4)	25 CNY, 50%	45 CNY, 25%	25 CNY, 50%	45 CNY, 25%
(5)	25 CNY, 50%	50 CNY, 25%	25 CNY, 50%	50 CNY, 25%
(6)	25 CNY, 50%	55 CNY, 25%	25 CNY, 50%	55 CNY, 25%
(7)	25 CNY, 50%	60 CNY, 25%	25 CNY, 50%	60 CNY, 25%

	Panel B: MPL for Question Series 2	
Question #	Choice X (Payoff, Prob)	Choice Y (Payoff, Prob)
(1)	(25 CNY, 50%; 20 CNY, 50%)	(30 CNY, 25%; 20 CNY, 75%)
(2)	(25 CNY, 50%; 20 CNY, 50%)	(35 CNY, 25%; 20 CNY, 75%)
(3)	(25 CNY, 50%; 20 CNY, 50%)	(40 CNY, 25%; 20 CNY, 75%)
(4)	(25 CNY, 50%; 20 CNY, 50%)	(45 CNY, 25%; 20 CNY, 75%)
(5)	(25 CNY, 50%; 20 CNY, 50%)	(50 CNY, 25%; 20 CNY, 75%)
(6)	(25 CNY, 50%; 20 CNY, 50%)	(55 CNY, 25%; 20 CNY, 75%)
(7)	(25 CNY, 50%; 20 CNY, 50%)	(60 CNY, 25%; 20 CNY, 75%)

Note: This table presents the content of the three groups of incentivized MPL questions. Panels A1 and C1 present the ROL for Question Groups 1 and 3, respectively. Names of colleges for the second, third, and fourth spot are replaced with safe colleges that students are familiar with. Panels A2 and C2 present the MPL for Question Groups 1 and 3, respectively. In both question groups, students need to select either College X or College Y from each row, and the selected college will be put at the first spot in the corresponding ROL. Panel B presents the MPL for Question Group 2. Similarly, students need to choose one from Lottery X and Y in each row. This table is helpful for the discussion in Section 6.1.

Table B5: Subjective Beliefs

	(1)	(2)	(3)	(4)
	Belief-Est. Prob.		| Belief-Est. Prob.|	
SES Index (Normalized)	1.20%**	2.17%***	1.05%***	0.93%**
	(0.51%)	(0.62%)	(0.35%)	(0.43%)
Mean	11.3%		30.6%	
Priority Score	Yes	Yes	Yes	Yes
Demographic Variables	No	Yes	No	Yes
College Preferences	No	Yes	No	Yes
Number of Observations	5341	5341	5341	5341
R squared	0.117	0.155	0.057	0.112

Note: This table presents analysis of student applicants' subjective beliefs about the unconditional admission probability for relevant colleges. All columns present regressions that examine whether subjective beliefs differ systematically from estimated admission probability, and whether such difference is correlated with SES Index. The outcome variable in Columns 1 and 2 is the difference between subjective beliefs and estimated admission probability. Columns 3 and 4 are the absolute difference between subjective beliefs and estimated admission probability. The mean of bias and absolute biases have been calculated below the coefficient estimates. Besides SES Index, the only additional covariate in Columns 1 and 3 is the fourth-order polynomial of priority score, whereas in Columns 2 and 4 we additionally control for demographics and stated college preferences. This table is helpful for the discussion in Section 9.1.

Table B6: Major Preferences and Its Impact on Risk Taking

	(1)	(2)	(3)	(4)	(5)	(6)
	Major Top Concern		Estimated Prob		Subjective Beliefs	
SES Index (Normalized)	-2.24%**	-2.66%**				
	(0.92%)	(1.17%)				
Major Top Concern			3.25%	4.09%	8.94%***	8.01%***
			(2.85%)	(3.00%)	(2.43%)	(2.51%)
Share of Major Top Concern	13.7%					
Implied Impact on 1st Choices			0.4%	0.6%	1.2%	1.1%
Priority Score	Yes	Yes	Yes	Yes	Yes	Yes
Demographic Variables	No	Yes	No	Yes	No	Yes
College Preferences	No	Yes	No	Yes	No	Yes
Number of Observations	1412	1412	1386	1386	1412	1412
R squared	0.009	0.091	0.138	0.201	0.035	0.087

Note: Columns 1 and 2 present regressions that examine whether share of survey takers who think major is their top concern is correlated with SES Index. Columns 3 and 4 present the regression that examines whether those who declare major to be of top concern take different amount of risks on their first choices, in terms of the estimated unconditional admission probability. Columns 5 and 6 examine whether those who declare major to be of top concern take different amount of risks on their first choices, in terms of subjective probability. Columns 1,3,5 only control for priority score, whereas Columns 2,4,6 additionally control for demographics as well as stated college preferences. The share of people who think major is most important have been reported below the estimate for Columns 1 and 2. The implied impact of major consideration on the risk-taking of first choices is reported below the estimated coefficient in Columns 3,4,5,6. This is calculated as the product of the share of people who think major is the most important, and the average of additional cautiousness for the first choices among this group of people. This table is helpful for the discussion in Section 9.3.

C Revealed Preference Implication of Reversals

In this section we detail the derivation described in Section 5.4. Let's begin with several basic notations.

- For any college A, the utility of admission is denoted by u_A. The admission probability for benchmark priority score s is denoted by p_A.
- The CDF of the cutoff of any single college A as a function of s is denoted by $\Phi(s - \mu_A)$, where μ_A measures the selectivity of college A.
- By definition, $p_A = \Phi(s - \mu_A)$. The shape of $\Phi(.)$ does not change with colleges because colleges that are not far from any specific student's priority score are assumed to have homoskadastic distribution of cutoffs.
- For a student whose priority score is s', the admission probability is denoted by p'_A. Similarly, p''_A when priority score is s'', p'''_A when priority score is s''', p''''_A for s'''' and so on. Same for other colleges.

Mathematically, for example, the following equation holds:

$$p'_A = \Phi(\Phi^{-1}(p_A) - s + s')$$

$$p''_A = \Phi(\Phi^{-1}(p_A) - s + s'')$$

$$p''_B = \Phi(\Phi^{-1}(p_B) - s + s'')$$

and so on.

First let's prove two lemmas to simplify the derivation.

Lemma 1 *Assume $\Phi(.)$ is continuous and log-concave. $s' < s$.*
If $p_A > p_B$, then

$$\frac{p_A}{p_B} < \frac{p'_A}{p'_B}$$

If $p_A < p_B$, then

$$\frac{p_A}{p_B} > \frac{p'_A}{p'_B}$$

Proof. Let

$$g(\delta) \equiv \frac{\Phi(\Phi^{-1}(p_A) - \delta)}{\Phi(\Phi^{-1}(p_B) - \delta)}$$

We have $g(0) = \frac{p_A}{p_B}$ and $g(s - s') = \frac{p'_A}{p'_B}$ Next we show that $g(\delta)$ is a monotone function:

$$\begin{aligned} g'(\delta) &= \frac{\Phi(\Phi^{-1}(p_A) - \delta)\Phi'(\Phi^{-1}(p_B) - \delta) - \Phi'(\Phi^{-1}(p_A) - \delta)\Phi(\Phi^{-1}(p_B) - \delta)}{\Phi(\Phi^{-1}(p_B) - \delta)^2} \\ &= \Phi(\Phi^{-1}(p_A) - \delta)\frac{\frac{\Phi'(\Phi^{-1}(p_B) - \delta)}{\Phi(\Phi^{-1}(p_B) - \delta)} - \frac{\Phi'(\Phi^{-1}(p_A) - \delta)}{\Phi(\Phi^{-1}(p_A) - \delta)}}{\Phi(\Phi^{-1}(p_B) - \delta)} \end{aligned}$$

As $\Phi(.)$ is log-concave, $\frac{d(ln(\Phi(x)))}{dx} = \frac{\Phi'(x)}{\Phi(x)}$ decreases over x.

When $p_A > p_B$, we have

$$\frac{\Phi'(\Phi^{-1}(p_B) - \delta)}{\Phi(\Phi^{-1}(p_B) - \delta)} > \frac{\Phi'(\Phi^{-1}(p_A) - \delta)}{\Phi(\Phi^{-1}(p_A) - \delta)}$$

which implies that

$$g'(\delta) > 0$$

Therefore

$$\frac{p'_A}{p'_B} = g(s - s') > g(0) = \frac{p_A}{p_B}$$

When $p_A < p_B$, we have

$$g'(\delta) < 0$$

Consequently,

$$\frac{p'_A}{p'_B} = g(s - s') < g(0) = \frac{p_A}{p_B}$$

■

Lemma 2 *Consider function*

$$\psi(t) = \frac{tf(t)}{k + tg(t)}$$

where $k > 0$, $t \geq 0$, $f(t) \geq 0$, $g(t) \geq 0$, $f'(t) \geq 0$. If $\frac{f(t)}{g(t)}$ weakly increases in t, then $\psi(t)$ increases in t.

Proof.

$$\psi'(t) = \frac{kf(t) + ktf'(t) + t^2[f'(t)g(t) - f(t)g'(t)]}{(k + tg(t))^2}$$

As $\frac{f(t)}{g(t)}$ increases in t, we have

$$\frac{f'(t)g(t) - f(t)g'(t)}{f^2(t)} \geq 0$$

Thus

$$\psi'(t) \geq \frac{kf(t) + ktf'(t)}{(k + tg(t))^2} \geq 0$$

■

Now we are ready to show the algebraic derivation of the results in Section 5.4. Throughout we assume that utility identifies a college. Outside option $\underline{u}$ is normalized to 0. If a ROL is shorter than 4, it means that students leave the rest of it blank.

Important notations before we derive the theorems:

- Let u denote utility vector $(u_1, u_2, .., u_n)$, preferences over colleges.
- The position of a college on the list is encoded by κ. $\kappa = 0$ if the college is omitted from the list; $\kappa = 1$ if the college is listed as the fourth choice; $\kappa = 2$ if the college is listed as the third

choice; $\kappa = 3$ if the college is listed as the second choice; $\kappa = 4$ if the college is listed as the first choice.

- Function $\mathcal{R} : (u, A, s) \mapsto \kappa$ maps utility u, college A, priority score s into position κ under Rational Decision Rule.
- Function $\mathcal{D} : (u, A, s) \mapsto \kappa$ maps utility u, college A, priority score s into position κ under DC Decision Rule.
- Set $\mathcal{C}^{RN}_{(u,A,\kappa)} = \{s|\mathcal{R}(u, A, s) = \kappa\}$ is the contour set of priority score s where A is listed as at position κ under Rational Decision Rule and preference u.
- Set $\mathcal{C}^{DC}_{(u,A,\kappa)} = \{s|\mathcal{D}(u, A, s) = \kappa\}$ is the contour set of priority score s where A is listed as at position κ under Rational Decision Rule and preference u.

Proposition 1 *Assume $\Phi(.)$ is continuous and log-concave. Given priority score $s' < s$, the following three scenarios (forms of upward movement) are impossible under Rational Decision Rule (if length of list is shorter than 4, it means that the rest is left blank):*

1. *Choose (x, y) when priority score is s'; choose (y, z) when priority score is s;*
2. *Choose (x, y, z) when priority score is s'; choose (y, z, w) when priority score is s;*
3. *Choose (x, y, z, w) when priority score is s'; choose (y, z, w, m) when priority score is s;*

Proof. Let's first prove Scenario 1 by contradiction, and then show that algebraically Scenario 2 reduces to Scenario 1, and Scenario 3 reduces to Scenario 2.

Scenario 1 The optimality implies that

$$p'_x u_x + (1 - p'_x)p'_y u_y \geq p'_y u_y + (1 - p'_y)p'_z u_z$$

and

$$p_x u_x + (1 - p_x)p_y u_y \leq p_y u_y + (1 - p_y)p_z u_z$$

Reorganize these two equations we get

$$\frac{p'_x}{p'_z} \geq \frac{1 - p'_y}{u_x - p'_y u_y} u_z$$

$$\frac{p_x}{p_z} \leq \frac{1 - p_y}{u_x - p_y u_y} u_z$$

As optimality also requires that $u_x \geq u_y \geq u_z$, we know that $p_x < p_z$. Lemma 1 thus implies that

$$\frac{p'_x}{p'_z} < \frac{p_x}{p_z}$$

On the other hand, let $t = 1 - p_y$, $f(t) = 1$, $g(t) = u_y$, $k = u_x - u_y$, then Lemma 2 applies to $\frac{1-p_y}{u_x - p_y u_y}$. This implies that the expression is increasing in t, and decreasing in p_y. Therefore,

$$\frac{1-p'_y}{u_x - p'_y u_y} u_z > \frac{1-p_y}{u_x - p_y u_y} u_z$$

which leads to contradiction.

Scenario 2 The optimality implies that

$$p'_x u_x + (1-p'_x)p'_y u_y + (1-p'_x)(1-p'_y)p'_z u_z \geq p'_y u_y + (1-p'_y)p'_z u_z + (1-p'_y)(1-p'_z)p'_w u_w$$

and

$$p_x u_x + (1-p_x)p_y u_y + (1-p_x)(1-p_y)p_z u_z \leq p_y u_y + (1-p_y)p_z u_z + (1-p_y)(1-p_z)p_w u_w$$

Reorganizing these two equations in a similar way we get

$$\frac{p'_x}{p'_w} \geq \frac{(1-p'_y)(1-p'_z)}{u_x - u_y + (1-p'_y)(u_y - p'_z u_z)} u_w$$

and

$$\frac{p_x}{p_w} \leq \frac{(1-p_y)(1-p_z)}{u_x - u_y + (1-p_y)(u_y - p_z u_z)} u_w$$

As $u_x > u_y > u_z > u_w$, we have $p_x < p_w$, so Lemma 1 can be applied again. Let $k = u_x - u_y$, $t = 1 - p_y$, $f(t) = 1 - p_z$, and $g(t) = u_y - p_z u_z$, it can be verified that these functions are appropriately defined. The remaining work is to check whether $\frac{f(t)}{g(t)}$ increases in t, a result we obtain in Scenario 1. So Lemma 2 applies.

Thus we obtain a similar contradiction.

Scenario 3 Similar to the manipulation above, we obtain

$$\frac{p'_x}{p'_m} \geq \frac{(1-p'_y)(1-p'_z)(1-p'_w)}{u_x - u_y + (1-p'_y)[u_y - p'_z u_z - (1-p'_z)p'_w u_w]} u_m$$

and

$$\frac{p_x}{p_m} \leq \frac{(1-p_y)(1-p_z)(1-p_w)}{u_x - u_y + (1-p_y)[u_y - p_z u_z - (1-p_z)p_w u_w]} u_m$$

Let $k = u_x - u_y$, $t = 1 - p_y$, $f(t) = (1-p_z)(1-p_w)$, and $g(t) = u_y - p_z u_z - (1-p_z)p_w u_w$, these functions are appropriately defined and we only need to verify that $\frac{(1-p_z)(1-p_w)}{u_y - p_z u_z - (1-p_z)p_w u_w}$ is an increasing function of t, which we have done in Scenario 2. Then we can obtain the same contradiction.

■

Proposition 2 *Assume $\Phi(.)$ is continuous and log-concave. If $\mathcal{R}(u, A, s) >= 1$, then for any $s' < s$, $\mathcal{R}(u, A, s') \geq \mathcal{R}(u, A, s)$ or $\mathcal{R}(u, A, s') = 0$.*

Proof. It suffices to show that $1 \leq \mathcal{R}(u, A, s') < \mathcal{R}(u, A, s)$ would lead to contradiction. Namely, it is impossible that college A is selected and appears at a lower position when priority score is lower.

Scenario 1 $\mathcal{R}(u, A, s) = 4$ and $\mathcal{R}(u, A, s') = 3$

This cannot possibly be both true. In other words, if a college is listed as the first choice, it cannot possibly be listed as the second choice when priority score is lower. To prove by contradiction, suppose the optimal list when priority score is s is (y, z, m, n) and the optimal list when priority score is $s' < s$ is (x, y, v, w), where m, n, v, w are just other colleges that could be identical or different. Let U and U' denote the expected utility of sub-portfolio (m, n), (v, w) respectively.

Apparently x and z cannot be the same college, which leads to contradiction immediately. The optimality implies that

$$p_x u_x + (1 - p_x) p_y u_y + (1 - p_x)(1 - p_y) U \geq p_y u_y + (1 - p_y) p_z u_z + (1 - p_y)(1 - p_z) U$$

Similarly,

$$p'_x u_x + (1 - p'_x) p'_y u_y + (1 - p'_x)(1 - p'_y) U' \leq p'_y u_y + (1 - p'_y) p'_z u_z + (1 - p'_y)(1 - p'_z) U'$$

Reorganizing these two inequalities, we have

$$\frac{p_x}{p_z} \leq \frac{(1 - p_y) u_z - (1 - p_y) U}{u_x - p_y u_y - (1 - p_y) U}$$

and

$$\frac{p'_x}{p'_z} > \frac{(1 - p'_y) u_z - (1 - p'_y) U'}{u_x - p'_y u_y - (1 - p'_y) U'}$$

The optimality implies that $u_x > u_y > u_z$. This in turn implies that $p_x < p_z$, because otherwise it is never optimal to list (y, z) as top two choices. When $v \neq z$ and $w \neq z$, U' cannot be larger than U, because this would imply that (v, w) is a better sub-portfolio than (m, n) when priority score is s.

From lemma 1 we have

$$\frac{p_x}{p_z} > \frac{p'_x}{p'_z}$$

However, we have

$$\begin{aligned}
&\frac{(1-p'_y)u_z-(1-p'_y)U'}{u_x-p'_yu_y-(1-p'_y)U'}\\
\geq&\frac{(1-p'_y)u_z-(1-p'_y)U}{u_x-p'_yu_y-(1-p'_y)U}\\
=&\frac{(1-p'_y)(u_z-U)}{u_x-p'_yu_y-(1-p'_y)U}\\
>&\frac{(1-p_y)(u_z-U)}{u_x-p_yu_y-(1-p_y)U}
\end{aligned}$$

which leads to contradiction.

When $v=z$, the list is (y,z,m,n) when priority score is s and (x,y,z,w) when priority score is s'. The optimality condition thus implies that $(y,z,m,n)\succ(x,y,z,n)$ when priority score is s, and $(x,y,z,w)\succ(y,z,m,w)$ when priority score is s'. In this case let U, U' denote the expected utility of n when priority score is s, w when priority score is s' respectively. From Proposition 1 we know that m cannot be w. Thus $U'\leq U$, as otherwise n would be suboptimal when score is s. From the preference ordering above (after similar algebraic manipulations) we have,

$$\frac{p'_x}{p'_m}\geq\frac{(1-p'_y)(1-p'_z)(u_m-U')}{u_x-u_y+(1-p'_y)[u_y-p'_zu_z-(1-p'_z)U']}$$

and

$$\frac{p_x}{p_m}\leq\frac{(1-p_y)(1-p_z)(u_m-U)}{u_x-u_y+(1-p_y)[u_y-p_zu_z-(1-p_z)U]}$$

Since $u_x>u_y>u_z>max\{u_w,u_m,u_n\}$, we have $p_x<p_m$. Thus from lemma 1 we know $\frac{p'_x}{p'_m}<\frac{p_x}{p_m}$. Also the expression on the right is decreasing in U, thus

$$\frac{(1-p'_y)(1-p'_z)(u_m-U')}{u_x-u_y+(1-p'_y)[u_y-p'_zu_z-(1-p'_z)U']}\geq\frac{(1-p'_y)(1-p'_z)(u_m-U)}{u_x-u_y+(1-p'_y)[u_y-p'_zu_z-(1-p'_z)U]}$$

Furthermore, recycling the manipulation trick in Proposition 1,

$$\frac{(1-p'_y)(1-p'_z)(u_m-U)}{u_x-u_y+(1-p'_y)[u_y-p'_zu_z-(1-p'_z)U]}\geq\frac{(1-p_y)(1-p_z)(u_m-U)}{u_x-u_y+(1-p_y)[u_y-p_zu_z-(1-p_z)U]}$$

which leads to contradiction.

When $w=z$, the list is (y,z,m,n) when priority score is s and (x,y,v,z) when priority score is s'. Optimality implies that $(v,z)\succ(z,m)$ when priority score is s', and $(z,m,n)\succ(v,z,n)$ when priority score is s. Mathematically, the latter is equivalent to

$$p_zu_z+(1-p_z)p_mu_m+(1-p_z)(1-p_m)p_nu_n\geq p_vu_v+(1-p_v)p_zu_z+(1-p_v)(1-p_z)p_nu_n$$

Since optimality also implies $u_x>u_y>u_v>u_z>u_m>u_n$, it implies that $p_m>p_v$. Thus $(z,m,n)\succ(v,z,n)$ implies that

$$p_z u_z + (1 - p_z) p_m u_m \geq p_v u_v + (1 - p_v) p_z u_z$$

which is equivalent to $(z, m) \succ (v, z)$. As Proposition 1 has proved, this choice pattern cannot be generated by the rational type.

Proof of Scenario 1 concludes.

Scenario 2 $\mathcal{R}(u, A, s) = 3$ and $\mathcal{R}(u, A, s') = 2$

In other words, a rational type chooses (m, y, z, n) when priority score is s; (v, x, y, w) when priority score is $s' < s$. When $w \geq z$, this case reduces to Scenario 1, where we can let U denote the expected utility of the last position and use exactly the same proof.

When $w = z$, $m \geq x$ because the choice pattern otherwise has been proved to be impossible in proposition 1. Thus the scenario implies that $(y, z, n) \succ (x, y, z)$ when priority score is s, but $(x, y, z) \succ (y, z, n)$ when priority score is s', an impossible pattern again.

Proof of Scenario 2 concludes.

Scenario 3 $\mathcal{R}(u, A, s) = 2$ and $\mathcal{R}(u, A, s') = 1$

In other words, a rational type chooses (m, n, y, z) when priority score is s; (v, w, x, y) when priority score is $s' < s$. If $m \neq x$ and $n \geq x$, the optimality condition requires that $(y, z) \succ (x, y)$ when score is s, but $(x, y) \succ (y, z)$ when score is s', which are impossible to hold at the same time.

If $n = x$, $m \neq w$ because of proposition 1. Consequently, $(x, y, z) \succ (w, x, y)$ when score is s, $(w, x, y) \succ (x, y, z)$ when score is s', again impossible thanks to Proposition 1.

If $m = x$, a rational type chooses (x, n, y, z) when priority score is s; (v, w, x, y) when priority score is $s' < s$. The optimality requires that when score is s', $(w, x, y) \succ (x, n, y)$, which is equivalent to

$$p'_w u_w + (1 - p'_w) p_x u_x + (1 - p'_w)(1 - p'_x) p'_y u_y > p'_x u_x + (1 - p'_x) p'_n u_n + (1 - p'_x)(1 - p'_n) p'_y u_y$$

The optimality also implies that $u_v > u_w > u_x > u_n > u_y > u_z$, and consequently $max\{p_v, p_w\} < min\{p_x, p_y, p_n, p_z\}$. As a result, we can infer that $(1 - p_w) > (1 - p_n)$, and consequently $(w, x, y, z) \succ (x, n, y, z)$ when score is s'.

Together with $(x, n, y, z) \succ (w, x, y, z)$ when score is s, this scenario becomes a standard case in Scenario 1.

Proof of Scenario 3 concludes.

Scenario 4 $\mathcal{R}(u, A, s) = 4$ and $\mathcal{R}(u, A, s') = 2$

In other words, the optimal list is (y, m, z, n) when score is s, (x, v, y, w) when score is s'. The optimality condition implies that $u_x > u_v > u_y > u_m > u_z > u_n$ and $u_y > u_w$, $max\{p_x, p_v\} < min\{p_y, p_m, p_z, p_n\}$.

When score is s, we have $(y, m, z, n) \succ (v, y, z, n)$. Mathematically,

$$p_y u_y + (1 - p_y) p_m u_m + (1 - p_y)(1 - p_m) EU[(z, n)] > p_v u_v + (1 - p_v) p_y u_y + (1 - p_v)(1 - p_y) EU[(z, n)]$$

If w is not m, the optimality condition when score is s implies that $(z, n) \succ (w)$, which is equivalent to $EU[(z, n)] > EU[(w)]$. As $(1 - p_m) < (1 - p_v)$, we have

$$p_y u_y + (1-p_y)p_m u_m + (1-p_y)(1-p_m)EU[(w)] > p_v u_v + (1-p_v)p_y u_y + (1-p_v)(1-p_y)EU[(w)]$$

which implies $(y, m, w) \succ (v, y, w)$ when score is s. On the other hand, $(v, y, w) \succ (y, m, w)$ when is score s'. As shown in Scenario 1, this is impossible.

If w is m, the optimal list is (y, w, z, n) when score is s, (x, v, y, w) when score is s'. In this case all the other colleges have to be different from each other: $u_x > u_v > u_y > u_w > u_z > u_n$. This in turn implies that $max\{p_l, p_W\} < min\{p_y, p_z, p_n, p_m\}$. The optimality condition when score is s' implies that $(x, y, z) \succ (y, z, n)$. The optimality condition when score is s implies that $(y, z, n, m) \succ (x, y, z, m)$. Mathematically:

$$EU[(y,z,n)] + (1-p_y)(1-p_z)(1-p_n)EU[(m)] \geq EU[(x,y,z)] + (1-p_x)(1-p_y)(1-p_z)EU[(m)]$$

As $(1-p_n) < (1-p_x)$, we have $EU[(y,z,n)] > EU[(x,y,z)]$, which implies that $(y, z, n) \succ (x, y, z)$ when score is s. Impossible according to Proposition 1.

Proof of Scenario 4 concludes.

Scenario 5 $\mathcal{R}(u, A, s) = 3$ and $\mathcal{R}(u, A, s') = 1$

In other words, the optimal list is (m, y, z, n) when score is s, and (x, v, w, y) when score is s'. The optimality condition requires that $u_x > u_v > u_w > u_y > u_z > u_n$ and $u_m > u_y$. This in turn implies that $max\{p_x, p_v, p_w\} < min\{p_m, p_y, p_z, p_n\}$. If all the letters here denote different colleges, the optimality condition implies that $(y, z, n) \succ (w, y, n)$ when score is s. As $(1-p_z) < (1-p_w)$, we have $(y, z) \succ (w, y)$ when score is s. When score is s', however, we have $(w, y) \succ (y, z)$, which is impossible under Rational Rule.

If some letters denote the same college, the only possibility is that m could be x, v or w. If $m = x$ or $m = v$, the same trick can be applied as well. If $m = w$, the optimal list is (w, y, z, n) when score is s, and (x, v, w, y) when score is s'. This case has actually been discussed in Scenario 4.

Proof of Scenario 5 concludes.

Scenario 6 $\mathcal{R}(u, A, s) = 4$ and $\mathcal{R}(u, A, s') = 1$

In other words, the optimal list is (y, m, n, l) when score is s, (w, v, x, y) when score is s'. The optimality condition implies that $u_w > u_v > u_x > u_y > u_m > u_n > u_l$, and $max\{p_w, p_v, p_x\} < min\{p_m, p_n, p_l, p_y\}$. As (w, v, x, y) is the optimal list when score is s', $(x, y) \succ (y, m)$ when score is s'.

Moreover, as the optimality list is (y, m, n, l) when score is s. We have $(y, m, n, l) \succ (x, y, n, l)$ when score is s. Mathematically this is equivalent to

$$p_y u_y + (1-p_y)p_m u_m + (1-p_y)(1-p_m)EU[(n,l)] \geq p_x u_x + (1-p_x)p_y u_y + (1-p_x)(1-p_y)EU[(n,l)]$$

As $(1-p_m) < (1-p_x)$, we have

$$p_y u_y + (1-p_y)p_m u_m \geq p_x u_x + (1-p_x)p_y u_y$$

which is equivalent to $(y, m) \succ (x, y)$. This pattern has been proved to be impossible in Proposition 1.

Proof of Scenario 6 concludes.

■

Proposition 3 *For any preference profile u, college A, $\mathcal{C}^{RN}_{(u,A,\kappa)} \equiv \{s|\mathcal{R}(u,A,s)=\kappa\}$ is connected if $\kappa \geq 1$.*

Proof.

Scenario 1 $\kappa = 4$

It suffices to show that if college x is listed in a specific position when priority score is s and s'', then it is the best candidate for that position as well for any s' such that $s'' < s' < s$. Mathematically

For any college $y \neq x$, we have

$$p_y u_y + (1-p_y)U \leq p_x u_x + (1-p_x)U$$

$$p''_y u_y + (1-p''_y)U'' \leq p''_x u_x + (1-p''_x)U''$$

where U represent the utility of the list of back-up colleges below the current position. Since $s'' < s' < s$, we have $U'' \geq U' \geq U$. Importantly, note that this holds because x cannot be any of the non-top choices ($1 \leq \mathcal{R}(u,A,s) < 4$) when score is s' thanks to Proposition 2.

The two equations above are equivalent to

$$\frac{p_x}{p_y} \geq \frac{u_y - U}{u_x - U}$$

$$\frac{p''_x}{p''_y} \geq \frac{u_y - U''}{u_x - U''}$$

Next we show that it is true that

$$\frac{p'_x}{p'_y} \geq \frac{u_y - U'}{u_x - U'}$$

We analyze whether x is a better choice when probability is p'_x, case by case. The first inequality in both case (I) and (II) is implied by Lemma 1.

Case (I): $p_x > p_y$, $u_x < u_y$. In this case we have

$$\frac{p'_x}{p'_y} \geq \frac{p_x}{p_y} \geq \frac{u_y - U}{u_x - U} \geq \frac{u_y - U'}{u_x - U'}$$

Case (II): $p_x < p_y$, $u_x > u_y$. In this case we have

$$\frac{p'_x}{p'_y} \geq \frac{p''_x}{p''_y} \geq \frac{u_y - U''}{u_x - U''} \geq \frac{u_y - U'}{u_x - U'}$$

Case (III): $p_x > p_y$, $u_x > u_y$, obviously x is better regardless of probability.

Case (IV): $p_x < p_y$, $u_x < u_y$, y must be chosen regardless of probability, which leads to contradiction.

Proof of Scenario 1 concludes.

Scenario 2 $\kappa \leq 3$

The proof of Scenario 1 can be largely recycled, with the only complication being whether x could be in a position where $\mathcal{R}(u, A, s) > \kappa$. This is again impossible thanks to Proposition 2.

■

Remark Proposition 2 and 3 together imply Theorem 1.

Proposition 4 *Suppose the cumulative distribution of cutoffs is log-concave and continuous, and for any s, the tie of expected utility among listed colleges is limited to at most two colleges. For any u and A, if there exists $s'' < s_0$ such that $\mathcal{D}(u, A, s'') = 0$ and $\mathcal{D}(u, A, s_0) = \kappa \geq 2$, then there exists $s'' < s' < s_0$ such that $1 \leq \mathcal{D}(u, A, s') \leq \kappa - 1$.*

Proof. Let $f_A(s) = p_A(s)u_A$. $f_A(s)$ is continuous for any college A. Thus for any pairs of A, x, function $\Delta_A x(s) \equiv f_A(s) - f_x(s)$ is continuous. According to lemma 1, $\Delta_{Ax}(s)$ switches signs at most once. If $p_A > p_x$, it can possibly switch from positive to negative; it $p_x > p_A$, it can possibly switch from negative to positive.

Define $\underline{s} \equiv sup_s\{s|\mathcal{D}(u, A, s) = 0\}$. According to the continuity we know that there exists college B such that $\Delta_{AB}(\underline{s}) = 0$. This college must be listed at the fourth place, and $\Delta_{AB}(s)$ is switching from negative to positive, because otherwise given the assumptions A will not be moved in the neighborhood of $\underline{s}$. Thus $\mathcal{D}(u, A, s) = 1$ in the neighborhood of $\underline{s}$.

If $\kappa = 3$, the key is to consider $\underline{s_2} \equiv sup_s\{s|\mathcal{D}(u, A, s) > 1\}$. We can infer using the same method that when s is in the neighborhood of s_2, $\mathcal{D}(u, A, s) = 2$.

If $\kappa = 4$, the key is to consider $\underline{s_3} \equiv sup_s\{s|\mathcal{D}(u, A, s) > 2\}$. We can infer using the same method that when s is in the neighborhood of s_3, $\mathcal{D}(u, A, s) = 3$.

■

Remark Proposition 4 implies Theorem 2.